MAC 2233
Calculus for Business
Graphing Calculator
Review # 1
E. Suco

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1. Use the graph shown in the figure to estimate the limits given:
(a) \( \lim_{x \to \infty} q(x) \)  (b) \( \lim_{x \to -\infty} q(x) \)  (c) \( \lim_{x \to 1} q(x) \)

1—1.4—ANSWER: (a) \( \lim_{x \to \infty} q(x) \to \infty \);  
(b) \( \lim_{x \to -\infty} q(x) \to -\infty \);  
(c) \( \lim_{x \to 1} q(x) \approx -2 \)

2. Use the graph shown in the figure to estimate the limits given:
(a) \( \lim_{t \to \infty} m(t) \)  (b) \( \lim_{t \to -\infty} m(t) \)  (c) \( \lim_{t \to 0} m(t) \)

1—1.4—ANSWER: (a) \( \lim_{t \to \infty} m(t) \approx 4 \);  
(b) \( \lim_{t \to -\infty} m(t) \to -\infty \);  
(c) \( \lim_{t \to 0} m(t) \approx 3.6 \)

3. Use the graph shown in the figure to estimate the limits given:
(a) \( \lim_{x \to -3^-} T(x) \)  (b) \( \lim_{x \to -3^+} T(x) \)  (c) \( \lim_{x \to -3} T(x) \)
(d) \( \lim_{x \to \infty} T(x) \)  (e) \( \lim_{x \to -\infty} T(x) \)

1—1.4—ANSWER: (a) \( \lim_{x \to -3^-} T(x) \to -\infty \);  
(b) \( \lim_{x \to -3^+} T(x) \to \infty \);  
(c) \( \lim_{x \to -3} T(x) \) does not exist;  
(d) \( \lim_{x \to \infty} T(x) = 0 \);  
(e) \( \lim_{x \to -\infty} T(x) = 0 \)

4. Use the graph shown in the figure to estimate the limits given:
(a) \( \lim_{t \to 0} R(t) \)  (b) \( \lim_{t \to 0^-} R(t) \)  (c) \( \lim_{t \to 0^+} R(t) \)
(d) \( \lim_{t \to \infty} R(t) \)  (e) \( \lim_{t \to -\infty} R(t) \)

1—1.4—ANSWER: (a) \( \lim_{t \to 0} R(t) \approx 1 \);  
(b) \( \lim_{t \to 0^-} R(t) \approx 3 \);  
(c) \( \lim_{t \to 0^+} R(t) \) does not exist;  
(d) \( \lim_{t \to \infty} R(t) \approx 3 \);  
(e) \( \lim_{t \to -\infty} R(t) \to \infty \)
5. A function \( h \) is such that \( \lim_{x \to 6^-} h(x) = 4.2 \), \( \lim_{x \to 6^+} h(x) = 4.2 \), and \( h(6) = 3 \).

(a) Does \( \lim_{x \to 6} h(x) \) exist? Explain.

(b) Is \( h \) continuous at \( x = 6 \)? Explain your reasoning.

(c) Sketch a possible graph of \( h \) near \( x = 6 \).

2. (a) Yes, \( \lim_{x \to 6} h(x) = 4.2 \) because the left and right limits equal this value. (b) No, \( h \) is not continuous at \( x = 6 \) because \( \lim_{x \to 6} h(x) \neq h(6) \). (c) See graph.

6. Use the graph of the function \( w \) that is shown in the figure to estimate each of the following limits. If any limit does not exist, indicate why not.

(a) \( \lim_{x \to 2^+} w(x) \)  
(b) \( \lim_{x \to 2^-} w(x) \)  
(c) \( \lim_{x \to 2} w(x) \)  
(d) \( \lim_{x \to 3^+} w(x) \)  
(e) \( \lim_{x \to 3^-} w(x) \)  
(f) \( \lim_{x \to -\infty} w(x) \)  
(g) \( \lim_{x \to \infty} w(x) \)  
(h) \( \lim_{x \to -\infty} w(x) \)

1. (a) \( \lim_{x \to 2^+} w(x) \to -\infty \); (b) \( \lim_{x \to 2^-} w(x) \to \infty \); (c) \( \lim w(x) \) does not exist because the left and right limits are not the same value; (d) \( \lim_{x \to 3^+} w(x) = 0 \); (e) \( \lim_{x \to 3^-} w(x) = 0 \); (f) \( \lim_{x \to -3} w(x) = 0 \); (g) \( \lim_{x \to \infty} w(x) \to \infty \); (h) \( \lim_{x \to -\infty} w(x) \to -\infty \)

7. Numerically estimate \( \lim_{x \to 25} \frac{\sqrt{x} - 5}{x - 25} \).

1. Let \( f(x) = \frac{\sqrt{x} - 5}{x - 25} \). It appears from the table given below that \( \lim_{x \to 25^+} f(x) \approx 0.1 \) and \( \lim_{x \to 25^-} f(x) \approx 0.1 \), so \( \lim_{x \to 25} f(x) \approx 0.1 \).

<table>
<thead>
<tr>
<th>( x \to 25^- )</th>
<th>( f(x) )</th>
<th>( x \to 25^+ )</th>
<th>( f(x) )</th>
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<tr>
<td>24.9</td>
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</tbody>
</table>
8. Numerically estimate \( \lim_{x \to 0.6} \frac{5x^2 + 17x - 12}{3 - 5x} \).

2—1.4—ANSWER: Let \( h(x) = \frac{5x^2 + 17x - 12}{3 - 5x} \). It appears from the table given below that

\[
\begin{array}{c|c|c|c}
\hline
x \to 0.6^- & h(x) & x \to 0.6^+ & h(x) \\
\hline
0.59 & 4.59 & 0.61 & 4.61 \\
0.599 & -4.599 & 0.601 & -4.601 \\
0.5999 & -4.5999 & 0.6001 & -4.6001 \\
0.59999 & -4.59999 & 0.60001 & -4.60001 \\
\hline
\end{array}
\]

9. Let \( g(x) = \frac{6x^2 - 19x + 10}{2x^2} \).

(a) Numerically estimate \( \lim_{x \to 2/3} g(x) \).

(b) Is the function \( g \) continuous at \( x = \frac{2}{3} \)? Explain.

2—1.4—ANSWER: (a) From the table given below, it appears that \( \lim_{x \to 2/3^-} g(x) \approx -3.67 \) and \( \lim_{x \to 2/3^+} g(x) \approx -3.67 \), so \( \lim_{x \to 2/3} g(x) \approx -3.67 \). (Note: Student \( x \)-values may vary from these shown but should approach \( \frac{2}{3} \) from the left and the right.) (b) The function \( g \) is not continuous at \( x = \frac{2}{3} \) because \( g\left(\frac{2}{3}\right) \) does not exist.

\[
\begin{array}{c|c|c|c}
\hline
x \to \left(\frac{2}{3}\right)^- & h(x) & x \to \left(\frac{2}{3}\right)^+ & h(x) \\
\hline
0.66 & -3.68 & 0.67 & -3.66 \\
0.666 & -3.668 & 0.667 & -3.666 \\
0.6666 & -3.6668 & 0.6667 & -3.6666 \\
0.66666 & -3.66668 & 0.66667 & -3.66666 \\
\hline
\end{array}
\]
The federal on-budget funds for all educational agencies (in constant 2000 dollars) between 1965 and 2000 can be described by the function

\[ f(t) = \begin{cases} 
0.011t^2 - 0.696t^2 + 14.380t - 29.370 & \text{billion dollars when } 5 \leq t < 35 \\
0.931t^2 - 67.374t + 1295.785 & \text{billion dollars when } 35 \leq t \leq 40 
\end{cases} \]

where \( t \) is the number of years past 1960. Is the function \( f \) continuous at \( t = 35 \)? Give, using limits, a reason for your answer. (Source: U.S. Department of Education, National Center for Educational Statistics)

2—1.4—ANSWER: Algebraically, we find that \( \lim_{t \to 35^-} f(t) = \lim_{t \to 35^-} (0.011t^2 - 0.696t^2 + 14.380t - 29.370) = 92.955 \) and \( \lim_{t \to 35^+} f(t) = \lim_{t \to 35^+} (0.931t^2 - 67.374t + 1295.785) = 78.17 \).

Therefore, because the left and right limits are not the same, \( \lim_{t \to 35} f(t) \) does not exist and \( f \) is not continuous at \( t = 35 \).

II.

Consider the function \( p(x) = \begin{cases} 
2x - 3 & \text{when } x \leq 0 \\
4x - 3 & \text{when } 0 < x \leq 2 \\
1 - \frac{1}{x} & \text{when } x > 2 
\end{cases} \)

(a) Find each of the following limits. If any limit does not exist, indicate why not.

i. \( \lim_{x \to 0^-} p(x) \)  ii. \( \lim_{x \to 0^+} p(x) \)  iii. \( \lim_{x \to 0^-} p(x) \)  iv. \( \lim_{x \to 2^-} p(x) \)

v. \( \lim_{x \to 2^+} p(x) \)  vi. \( \lim_{x \to -\infty} p(x) \)  vii. \( \lim_{x \to +\infty} p(x) \)  viii. \( \lim_{x \to +\infty} p(x) \)

(b) Is \( p \) continuous at \( x = 0 \)? Explain.
(c) Is \( p \) continuous at \( x = 2 \)? Explain.
(d) Give the equations of any lines that are horizontal asymptotes for the function \( p \).

2—1.4—ANSWER: (a) i. \( \lim_{x \to 0^-} p(x) = -3 \), ii. \( \lim_{x \to 0^+} p(x) \to -\infty \), iii. \( \lim_{x \to 0^-} p(x) \) does not exist because the left and right limits are not the same. iv. \( \lim_{x \to 2^-} p(x) = 0.5 \), v. \( \lim_{x \to 2^+} p(x) = 0.5 \),

vi. \( \lim_{x \to 2^-} p(x) = 0.5 \), vii. \( \lim_{x \to +\infty} p(x) \to -\infty \), viii. \( \lim_{x \to +\infty} p(x) = 1 \); (b) No, \( p \) is not continuous at \( x = 0 \) because \( \lim_{x \to 0^-} p(x) \) does not exist. (c) Yes, \( p \) is continuous at \( x = 2 \) because \( \lim_{x \to 2^-} p(x) = 0.5 \) and \( p(2) = \frac{4(2)^2 - 3}{5(2)} = 0.5 \). (d) A horizontal asymptote for \( p \) is the line \( y = p(x) = 1 \).

III.

Algebraically find \( \lim_{t \to 16} \frac{t - 16}{4\sqrt{t} - 16} \). (Hint: Multiply numerator and denominator by \( \sqrt{t} + 4 \).)

2—1.4—ANSWER: \( \frac{t - 16}{4\sqrt{t} - 16} \cdot \frac{\sqrt{t} + 4}{\sqrt{t} + 4} = \frac{(t - 16)(\sqrt{t} + 4)}{4(t - 16)} = \frac{\sqrt{t} + 4}{4} \) for \( t \neq 16 \).

So, \( \lim_{t \to 16} \frac{t - 16}{4\sqrt{t} - 16} = \lim_{t \to 16} \frac{\sqrt{t} + 4}{4} = 2 \).
13. For each of the following:
   i. Find the slope.
   ii. Find the y-value of the vertical-axis intercept.
   iii. Find the x-value of the horizontal axis intercept.
   iv. State whether the line is rising or falling.

Finally, rank the three lines in order of increasing steepness.

(a) \( y = 3 - 2.5x \)  
(b) \( y = 0.01x - 2.53 \)  
(c) \( y = 1.3x + 7 \)

1—1.5—ANSWER: (a) i. Slope = -2.5, ii. \( y = 3 \), iii. \( x = \frac{3}{2.5} \) or \( x = 1.2 \), iv. Falling;  
(b) i. Slope = 0.01, ii. \( y = -2.53 \), iii. \( x = 253 \), iv. Rising; (c) i. Slope = 1.3, ii. \( y = 7 \),  
iii. \( x = \frac{-7}{1.3} \) or \( x \approx -5.38 \), iv. Rising; Line b, Line c, Line a

14. The average mobile-phone local monthly bill in the U.S. between 1995 and 1998 is modeled by \( B(x) = -3.963x + 51.172 \) dollars, where \( x \) is the number of years after 1995. (Source: Based on data appearing in Newsweek, March 20, 2000)

(a) Find and interpret the vertical axis intercept.
(b) What is the rate of change of \( B \)?
(c) Predict when the average local monthly bill was $35.32. How reliable is this prediction?

1—1.5—ANSWER: (a) Set \( x = 0 \) to find \( B(0) = 51.172 \). A possible interpretation is: The average mobile-phone local monthly bill in the U.S. in 1995 was about $51.72. (b) About -3.96 dollars per year; (c) Set \( B(x) = 35.32 \) to find \( x \approx 4 \), so the answer is approximately 1999. This input is just one year outside the time interval within which the model is valid, so it might be a reasonable prediction.

15. The function \( T(x) = 20.37 + 1.834x \) hundred dollars, where \( x \) is the number of years past 1980, describes the per capita tax burden in the U.S. between 1980 and 1999. (Source: Internal Revenue Service)

(a) Find the horizontal axis intercept. Does this value have a valid interpretation in the context?
(b) Find and interpret the vertical axis intercept.
(c) Sketch a graph of \( T \).
(d) What is the rate of change of \( T \)?

1—1.5—ANSWER: (a) Set \( T(x) = 0 \) to find \( x \approx -11.11 \). No, this input corresponds to about 1969, which is not within the interval of time for which the model is valid. Also, the per capita tax burden in the U.S. definitely was not $0 in 1969. (b) Set \( x = 0 \) to find \( T(0) = 20.37 \). A possible interpretation is: The per capita tax burden in the U.S. in 1980 was $2037. (c) See graph below. (d) 1.834 hundred dollars per year or $183.40 per year
16. Fast-food spending in the USA between 1985 and 1993 was approximately linear, as shown in the figure. (Source: USA Today, June 20, 1994, p. D1)
(a) How many did Americans spend on fast food in 1990?
(b) Estimate the rate of change of fast-food spending between 1985 and 1993.
(c) Interpret your answer to part b.
(d) Would the horizontal axis intercept have meaning in this context? Explain.

1 –1.5—ANSWER: (a) Approximately $83.6 billion; (b) About 3.6 billion dollars per year; (c) Fast-food spending in the U.S. was increasing by about $3.6 billion per year during the 8-year time period. (d) Answers will vary.

17. A calculus professor observes that the number of days a student skips class is approximately linearly related to that student's chance of failing the course as shown in the graph.
(a) Estimate the slope of the graph, and write a sentence explaining the meaning of the slope in this context.
(b) What is the rate of change of the student's chance of failing the course?
(c) Identify the vertical axis intercept, and interpret its meaning in this context.

2—1.5—ANSWER: (a) slope ≈ 2, A student's chance of failing the course is increasing by about 2 percentage points for each class that is cut (b) 2 percentage points per class cut; (c) The graph crosses the vertical axis at the point where the chance of failing is approximately 44%. A possible interpretation is: A student has about a 44% chance of failing the calculus course when that student does not cut any classes.

18. The percentage of taxpayers who contribute to the presidential election fund fell at an almost constant rate between 1980 and 1994. Data from the Federal Election Commission indicate that 28.7% of taxpayers contributed to this fund in 1980 and 13.0% contributed in 1994.
(a) How fast was the percentage of contributing taxpayers falling between 1980 and 1994?
(b) Find an equation for the percentage of contributing taxpayers in terms of the year.
(c) Find an equation for the percentage of contributing taxpayers in terms of the number of years after 1980.
(d) Find an equation for the percentage of contributing taxpayers in terms of the number of years after 1900.
(e) What percent of taxpayers contributed to the presidential election fund in 1990?

1—1.5—ANSWER: (a) The percentage was falling at a rate of about 1.1 percentage points per year; (b) \( f(x) = -1.1214x + 2249.1286 \) percent of taxpayers contributing in year \( x \); (c) \( p(y) = -1.1214y + 28.7 \) percent of taxpayers contributing \( y \) years after 1980, (d) \( c(u) = 1.1214u + 118.4143 \) percent of taxpayers contributing \( u \) years after 1900; (e) \( t(1990) = p(10) = c(90) \approx 17.5\% \) of taxpayers
The U.S. per capita consumption of asparagus between 1990 and 1996 is shown in the table.

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</thead>
<tbody>
<tr>
<td>Asparagus consumption (pounds per person)</td>
<td>0.6</td>
<td>0.6</td>
<td>0.6</td>
<td>0.6</td>
<td>0.6</td>
<td>0.6</td>
<td>0.6</td>
</tr>
</tbody>
</table>

(Source: Statistical Abstract, 1998)

(a) Find a linear model for the consumption in terms of the number of years since 1990.
(b) Sketch a graph of the function.
(c) What is the slope of the graph? Interpret the meaning of the slope in this context.
(d) Identify and explain the significance in this context of the vertical axis intercept of the graph in part b.
(e) If you found a linear model for the consumption in terms of the year, would the vertical axis intercept have a valid interpretation in this context?
(f) What is the rate of change of asparagus consumption between 1990 and 1996?

1—1.5—ANSWER: (a) \( A(t) = 0.6 \) pound per person of asparagus consumed between 1990 and 1996, where \( t \) is the number of years after 1990; (b) See graph below. (c) Slope = 0; Per capita asparagus consumption was constant between 1990 and 1996. (d) The graph in part b crosses the vertical axis at the point where \( A(t) = 0.6 \). This value is the per capita asparagus consumption in 1990. (e) The horizontal line would cross the vertical axis at the point (0, 0.6). The zero input could only represent the per-capita asparagus consumption in year 0, but this value has no meaning because it is too far to the left of the given data interval. (f) 0 pounds/person/year

The SAT mean math score of college-bound seniors from 1994 through 1996 can be modeled by the equation \( y = 2x + 504 \) points where \( x \) is the number of years after 1994. (Source: Based on data from The College Board)

(a) What is the slope of a graph of the equation?
(b) What is the rate of change of the SAT mean math score?
(c) Sketch an accurate graph of the SAT mean math score over this period.

1—1.5—ANSWER: (a) 2 points per year; (b) 2 points per year; (c) See graph below.
21. The graph shows the number of newspapers sold each month at a certain newsstand over a period of time.
   (a) Estimate the slope of the graph, and explain its meaning in this context.
   (b) What is the rate of change of the number of newspapers sold during this time period?
   (c) What is the meaning of the horizontal axis intercept for this graph?

1—1.5—ANSWER: (a) Approximately -0.2 thousand newspapers per month; The monthly sale of newspapers was declining by about 200 newspapers during the five-month period; (b) About -0.2 thousand newspapers per month (or -200 newspapers per month); (c) The input value of the point where this graph crosses the horizontal axis gives the time when the number of newspapers sold each month is 0.

22. The total production of hay by farms in the United States from 1993 through 1995 is given by $y = 4x + 146.533$ million tons where $x$ is the number of years after 1993. (Source: Based on data from the U.S. Department of Agriculture)
   (a) What is the rate of change of the number of tons of hay produced in the United States from 1993 through 1995?
   (b) What is the slope of a graph of the linear equation?
   (c) How much hay was produced in 1995?

1—1.5—ANSWER: (a) 4 million tons per year; (b) 4 million tons per year; (c) About 154.533 million tons

23. In the game of Monopoly® that was invented in 1935, the rent fee for Boardwalk is $50. When adjusted for inflation, this rent was equivalent to $610.28 in 1995. (Source: Based on data from Parker Brothers)
   (a) Find the rate of change of the value of rent for Boardwalk from 1935 to 1995.
   (b) Assume that the rate of increase remained constant and complete the table.
   (c) Find an equation for the value of rent for Boardwalk in terms of the year.

1—1.5—ANSWER: (a) $9.338 per year; (b) $50, $143.38, $236.76, $330.14, $423.52, $516.90, $610.28; (c) Boardwalk rent = 9.338x + 50 dollars where $x$ is the number of years since 1935.
24. The years in the left column of a table of data are 1970, 1975, 1980, 1985, 1990, and 1995. How would you describe the input $x$ of a function that models data for these years if the aligned input used to find the equation is as given in each of the following?
(b) 0, 5, 10, 15, 20, 25
(c) 11, 16, 21, 26, 31, 36
(d) −30, −25, −20, −15, −10, −5

1—1.5—ANSWER: (a) $x$ is the year; (b) $x$ is the number of years after 1970; (c) $x$ is the number of years after 1959; (d) $x$ is the number of years after 2000

25. A function $R$ with input $t$ models the revenue of a small business. The output of $R$ is measured in thousands of dollars.
(a) Write this sentence using the indicated input description and the appropriate mathematical symbols: “The revenue of the business in 1990 was $85,000.”
   i. $t$ is the number of years after 1988
   ii. $t$ is the year
   iii. $t$ is the number of decades (i.e., 10-year time periods) after 1900
(b) If $t$ is the number of years after 1990, write a sentence interpreting $R(11) - R(9) = -2.69$.

1—1.5—ANSWER: (a) i. $R(2) = 85$; ii. $R(1990) = 85$; iii. $R(9) = 85$; (b) The revenue of the small business decreased $2690$ between 1999 and 2001.

26. For the given input data of a function whose input variable is $x$, write the aligned input for each of the following descriptions.
(a) Input values for time: 1972, 1982, 1992, 2002
   i. $x$ is the number of years after 1970
   ii. $x$ is the number of years after 1900
(b) Input values for height: 5 feet 9 inches, 6 feet, 6 feet 3 inches, 6 feet 6 inches
   i. $x$ is the height in inches
   ii. $x$ is the height (in feet) in excess of 5 feet

1—1.5—ANSWER: (a) i. Aligned input: 2, 12, 22, 32; ii. Aligned input: 72, 82, 92, 102;
   (b) i. Aligned input: 69, 72, 75, 78; ii. Aligned input: 0.75, 1, 1.25, 1.5
27. The table shows the total amount paid by Americans for personal taxes from 1991 through 1994. (Source: Bureau of Economic Analysis)

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<tr>
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<tbody>
<tr>
<td>Taxes (billions of dollars)</td>
<td>624.8</td>
<td>650.5</td>
<td>689.9</td>
<td>731.4</td>
</tr>
</tbody>
</table>

(a) Find a linear model for these data.
(b) What is the rate of change of the amount paid for personal taxes?
(c) Estimate the amount paid for taxes in 1990 and in 1995.

1—1.5—ANSWER: (a) $T(x) = 35.92x + 620.27$ billion dollars $x$ years after 1991; (b) 35.92 billion dollars per year; (c) $T(-1) = \$584.35$ billion, $T(4) = \$763.95$ billion

28. (a) Match each graph with its equation.

\[ f(x) = 4(0.37^x) \]
\[ g(x) = 4(1.06^x) \]

(b) Which function, $f$ or $g$, represents exponential growth?

1—2.1—ANSWER: (a) Graph $i$ is a graph of $f$. Graph $ii$ is a graph of $g$. (b) Function $g$ is an exponential growth function.

29. (a) Match each graph with its equation.

\[ f(x) = -1.9(0.86^x) \]
\[ g(x) = 1.9(0.86^x) \]

(b) Which function, $f$ or $g$, represents exponential decay?

1—2.1—ANSWER: (a) Graph $i$ is a graph of $g$. Graph $ii$ is a graph of $f$. (b) Function $g$ is an exponential decay function.

30. Match each graph with its equation.

\[ f(x) = 1.6 \ln x \]
\[ g(x) = -1.6 \ln x \]
\[ h(x) = -1.6e^x \]

1—2.1—ANSWER: Graph $a$ is a graph of $g$. Graph $b$ is a graph of $h$. Graph $c$ is a graph of $f$. 
31. Match each graph with its equation.

\[ f(x) = 2(0.5^x) \]  
\[ g(x) = 2(1.5^x) \]  
\[ h(x) = 1.5 \ln x \]

1—2.1—Answer: Graph a is a graph of \( h \). Graph b is a graph of \( f \). Graph c is a graph of \( g \).

32. Let \( g(y) = 12.63(1.37^y) \).

(a) Find \( g(7) \).
(b) Does this function describe exponential growth or decay?
(c) Is the function increasing or decreasing? How can you tell without graphing the function?
(d) What is the constant percentage change? Explain.

1—2.1—Answer: (a) \( g(7) \approx 114.406 \); (b) Exponential growth; (c) The function is increasing because \( b = 1.37 > 1 \). (d) The constant percentage change is \( (b - 1)100\% = (1.37 - 1)100\% = 37\% \), so \( g(y) \) is increasing with a 37% change in output for every unit of input.

33. Let \( k(x) = 0.86(1.03^x) \).

(a) Find \( k(2) \).
(b) Does this function describe exponential growth or decay?
(c) Is the function increasing or decreasing? How can you tell without graphing the function?
(d) What is the constant percentage change? Explain.

1—2.1—Answer: (a) \( k(2) \approx 0.912 \); (b) Exponential growth; (c) The function is increasing because \( b = 1.03 > 1 \). (d) The constant percentage change is \( (b - 1)100\% = (1.03 - 1)100\% = 3\% \), so \( k(t) \) is increasing with a 3% change in output for every unit of input.

34. Let \( m(x) = 0.92(0.64^x) \).

(a) Find \( m(10) \).
(b) Does this function describe exponential growth or decay?
(c) Is the function increasing or decreasing? How can you tell without graphing the function?
(d) What is the constant percentage change? Explain.

1—2.1—Answer: (a) \( m(10) \approx 0.011 \); (b) Exponential decay; (c) The function is decreasing because \( b = 0.64 < 1 \); (d) The constant percentage change is \( (b - 1)100\% = (0.64 - 1)100\% = -36\% \), so \( m(x) \) is decreasing with a 36% change in output for every unit of input.

35. Let \( P(a) = 100(0.871^a) \).

(a) Find \( P(3) \).
(b) Is the function increasing or decreasing? How can you tell without graphing the function?
(c) What is the constant percentage change? Explain.

1—2.1—Answer: (a) \( P(3) \approx 66.078 \); (b) The function is decreasing because \( b = 0.871 < 1 \); (c) The constant percentage change is \( (b - 1)100\% = (0.871 - 1)100\% = -12.9\% \), so \( P(a) \) is decreasing with a 12.9% change in output for every unit of input.
35. The total number of hours spent on line by U.S. residents during 1994 can be described by the function \( H(m) = 12(1.12^m) \) million hours by the end of the \( m \)th month after the beginning of January 1994.
   (a) Was the total number of hours spent on line growing or decaying during 1994?
   (b) Find the constant percentage change for this situation.
   (c) Interpret the constant percentage change in this context.

1—2.1—ANSWER: (a) Growing; (b) 12%; (c) The total number of hours spent on-line by U.S. residents increased by 12% each month during 1994.

36. The number of cases of a certain infectious disease reported in the United States can be modeled by \( D(t) = 225(0.743^t) \) thousand cases after \( t \) years.
   (a) How many cases of the disease were reported initially?
   (b) Find the constant percentage change for this situation.
   (c) Interpret the constant percentage change in this context.

1—2.1—ANSWER: (a) \( D(0) = 225,000 \) cases; (b) \(-25.7\%\); (c) The number of cases reported declines by 25.7% each year.

37. The table shows the percent of traffic accidents investigated by the Ohio State Highway Patrol that were fatal crashes from 1991 through 1995. (Source: Ohio State Highway Patrol)

<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>Fatal crashes (percent)</td>
<td>0.93726</td>
<td>0.79979</td>
<td>0.78662</td>
<td>0.70377</td>
<td>0.67086</td>
</tr>
</tbody>
</table>

(a) Fit an exponential model to the data in the table.
(b) Assuming the trend in these data applies to years outside the interval 1991 through 1995, when was the percent of fatal crashes 1.063%?

2—2.1—ANSWER: (a) \( F(t) = 0.908(0.9234^t) \) percent \( t \) years after 1991; (b) \( t \approx -1.98 \); The percent of fatal crashes was 1.063%, and this occurred near the end of 1989.

38. The mouse population in the subways of a certain city is estimated at 8,000. Rodent control experts estimate that the population grows by 4% each month.
   (a) Find a model for the number of mice living in the subways.
   (b) Predict the mouse population in 9 months.
   (c) If rodent control experts decide to begin a control program when the population reaches 12,000 mice, in how many months will they need to start this program?

1—2.1—ANSWER: (a) \( P(x) = 8000(1.04^x) \) mice after \( x \) months; (b) \( P(9) = 12,000 \) mice;
(c) Solve \( P(x) = 12,000 \) for \( x \) to find \( x \approx 20.84 \). Because \( P \) should be interpreted literally, the program should be started in about 21 months.

39. A toy that was expected to be a passing fad had sales of $40 million in its first year, then declined by 32% each year.
   (a) Find a model for the sales of the toy.
   (b) Estimate the toy's sales 3 years later.

1.2.1—ANSWER: (a) \( S(t) = 40(0.68^t) \) million dollars \( t \) years after the
(b) \( S(3) \approx 12.577 \) million
One of the United Nations projections for world population growth between 1995 and 2150 is as shown in the table. (Source: Department of Economic and Social Affairs, United Nations)

<table>
<thead>
<tr>
<th>Year</th>
<th>Projected population (in billions)</th>
<th>Year</th>
<th>Projected population (in billions)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1995</td>
<td>5666</td>
<td>2075</td>
<td>26,048</td>
</tr>
<tr>
<td>2000</td>
<td>6113</td>
<td>2100</td>
<td>52,508</td>
</tr>
<tr>
<td>2025</td>
<td>9069</td>
<td>2125</td>
<td>113,302</td>
</tr>
<tr>
<td>2050</td>
<td>14,421</td>
<td>2150</td>
<td>255,846</td>
</tr>
</tbody>
</table>

(a) Find an exponential function for these data. Use as input the number of years after 1990.
(b) What percentage growth does the function in part a represent?
(c) Determine the outputs from the function in 2000, 2050, 2100, and 2150. Compare these values with the values in the table.

2—2.1—ANSWER: (a) \( P(t) = 4154.8718(1.0245^t) \) billion people where \( t \) is the number of years after 1990; (b) 2.45% growth rate; (c) See table below. The 2000 and 2150 values are underestimates, but the 2050 and 2100 outputs are overestimates.

<table>
<thead>
<tr>
<th>Year</th>
<th>Projected population (in billions)</th>
<th>( P(t) ) output (in billions)</th>
</tr>
</thead>
<tbody>
<tr>
<td>2000</td>
<td>6113</td>
<td>5292</td>
</tr>
<tr>
<td>2050</td>
<td>14,421</td>
<td>17,748</td>
</tr>
<tr>
<td>2100</td>
<td>52,508</td>
<td>59,515</td>
</tr>
<tr>
<td>2150</td>
<td>255,846</td>
<td>199,579</td>
</tr>
</tbody>
</table>

(a) Find an exponential function for the sales of prepaid phone calling cards.
(b) What percentage growth rate does the function in part a represent?
(c) When did 1992 sales triple in value?

1—2.1—ANSWER: (a) \( S(t) = 2.2182(2.3259^t) \) million dollars where \( t \) is the number of years after 1990; (b) A growth rate of about 123.6%; (c) Solve \( S(t) = 36 \) to find \( t \approx 3.3 \). In 1995 the sales had not yet tripled, so the answer is 1996.
12.a) Identify the scatter plot in the figure as linear, exponential, log, logistic, or none of these. Give reasons for your choice.

1--2.3—ANSWER: These data would be best modeled by a linear function because the points appear to fall along a straight line.

b) Identify the scatter plot in the figure as linear, exponential, log, logistic, or none of these. Give reasons for your choice.

1--2.3—ANSWER: These data would be best modeled by an exponential function because the plot is curved, approaches the horizontal axis, and does not change concavity.

c) Identify the scatter plot in the figure as linear, exponential, log, logistic, or none of these. Explain your reasoning.

1--2.3—ANSWER: These data cannot be modeled by any of these models. The plot exhibits a change in concavity, so it cannot be modeled by a linear, exponential, or log function. The plot also does not level off like a logistic model.

d) Identify the scatter plot in the figure as linear, exponential, log, logistic, or none of these. Explain your reasoning.

1--2.3—ANSWER: These data would be best modeled by an exponential model because the plot is curved, approaches the horizontal axis, and does not change concavity.

e) Identify the scatter plot in the figure as linear, exponential, log, logistic, or none of these. Give reasons for your choice.

1--2.3—ANSWER: These data would be best modeled by a logistic function because the plot shows a change in concavity and appears to begin leveling off in both directions.
Identify the scatter plot in the figure as linear, exponential, log, logistic, or none of these. Give reasons for your choice.

1—2.3—ANSWER: These data would be best modeled by a log function because the plot shows a single concavity and rises slowly after an initial rapid rise.

Let \( G(w) = \frac{44.6}{1 + 13.37e^{-4.7w}} \).

(a) Find \( G(1) \).
(b) Is the function increasing or decreasing? How can you tell?
(c) What is the limiting value of the function? Explain.
(d) Give the equations of the two horizontal asymptotes.

1—2.3—ANSWER: (a) \( G(1) \approx 39.764 \); (b) Increasing because \( B = 4.7 > 0 \); (c) The limiting value is 44.6. This means that as \( x \) becomes larger and larger, the value of the function becomes closer and closer to 44.6. (d) \( y = 0 \) and \( y = 44.6 \)

Let \( r(x) = \frac{222}{1 + 152.56e^{-0.328x}} \).

(a) Find \( r(10) \).
(b) Is the function increasing or decreasing? How can you tell?
(c) What is the limiting value of the function? Explain.
(d) Give the equations of the two horizontal asymptotes.

1—2.3—ANSWER: (a) \( r(10) \approx 32.935 \); (b) Increasing because \( B = 0.328 > 0 \); (c) The limiting value is 222. This means that as \( x \) becomes larger and larger, the value of the function becomes closer and closer to 222. (d) \( y = 0 \) and \( y = 222 \)

Let \( Q(t) = \frac{66.16}{1 + 0.040e^{0.0223t}} \).

(a) Find \( Q(5) \).
(b) Is the function increasing or decreasing? How can you tell?
(c) What is the limiting value of the function? Explain.

1—2.3—ANSWER: (a) \( Q(5) \approx 58.967 \); (b) Decreasing because \( B = -0.223 < 0 \); (c) The limiting value is 66.16. This means that as \( x \) becomes larger and larger, the function output becomes closer and closer to 66.16.

Let \( s(c) = \frac{10,100}{1 + 0.0077e^{0.254c}} \).

(a) Find \( s(13) \).
(b) Is the function increasing or decreasing? How can you tell?
(c) What is the limiting value of the function? Explain.

1—2.3—ANSWER: (a) \( s(13) \approx 8352.7 \); (b) Decreasing because \( B = -0.254 < 0 \); (c) The limiting value is 10,100. This means that the value of the function is never more than 10,100.
47. The table shows the number of bank failures in the United States from 1988 through 1995. (Source: Federal Deposit Insurance Corporation)

<table>
<thead>
<tr>
<th></th>
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<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>Failures</td>
<td>221</td>
<td>207</td>
<td>169</td>
<td>127</td>
<td>41</td>
<td>13</td>
<td>6</td>
</tr>
</tbody>
</table>

(a) Examine a scatter plot of the data. Is the plot increasing or decreasing? Does there appear to be an inflection point?
(b) Find a logistic model to fit the data. How well does the model fit?
(c) In approximately what year does the inflection point appear?

2—2.3—ANSWER: (a) The scatter plot is decreasing, and it does exhibit an inflection point; (b) \( F(x) = \frac{234.779}{1 + 0.061 e^{-0.889 x}} \) failures x years after 1988; the model is a good fit; (c) The inflection point appears to occur when the input is about 1992.

48. The table shows the percent of U.S. households that owned a microwave oven for selected years from 1978 through 1993. (Source: Energy Information Administration)

<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>Households (percent)</td>
<td>8</td>
<td>14</td>
<td>21</td>
<td>34</td>
<td>61</td>
<td>79</td>
<td>84</td>
</tr>
</tbody>
</table>

(a) Find a logistic model for the data. How well does the model fit?
(b) Sketch the graph of the model. Label the inflection point on the graph. Where is the curve concave up and where is it concave down?

2—2.3—ANSWER: (a) \( P(x) = \frac{90.275}{1 + 13.41 e^{-0.3649 x}} \) percent of U.S. households owning a microwave x years after 1978; the model gives a good fit; (b) See graph. The input value of the inflection point appears to be about 1985. The curve is concave up to the left of the inflection point and concave down to the right of the inflection point.

49. Suppose that between 1980 and 2000 the size of a small town changed according to the equation

\[ P(t) = \frac{13.70}{1 + 20.17 e^{-0.25 t}} \text{ thousand people where } t \text{ is the number of years after 1980.} \]

(a) Was the population of the town increasing or decreasing between 1980 and 2000?
(b) Find and interpret \( P(10) \).
(c) Does there appear to be an upper limiting value for the size of the population? If so, what is this value?

2—2.3—ANSWER: (a) Increasing; (b) \( P(10) \approx 5.16 \). There were about 5160 people living in this small town in 1990. (c) Yes, according to the model, the population will never exceed 13,700 people.
50. a) Classify the graph as concave up or concave down. Over which portions of the horizontal axis is the curve increasing or decreasing?

1—2.4—ANSWER: Concave up and always increasing

b) Classify the graph as concave up or concave down. Over which portions of the horizontal axis is the curve increasing or decreasing?

1—2.4—ANSWER: Concave up and always decreasing

c) Classify the graph as concave up or concave down. Over which portions of the horizontal axis is the curve increasing or decreasing?

1—2.4—ANSWER: Concave down; increasing from 0 to about 3 and decreasing from 3 to approximately 7.9

d) Classify the graph as concave up or concave down. Over which portions of the horizontal axis is the curve increasing or decreasing?

1—2.4—ANSWER: Concave up; decreasing from -2 to about 4 and increasing from 4 to 10
Refer to the table below.

<table>
<thead>
<tr>
<th>x</th>
<th>0</th>
<th>2</th>
<th>4</th>
<th>6</th>
<th>8</th>
<th>10</th>
<th>12</th>
<th>14</th>
<th>16</th>
</tr>
</thead>
<tbody>
<tr>
<td>y</td>
<td>22.5</td>
<td>9.3</td>
<td>2.1</td>
<td>0.9</td>
<td>5.7</td>
<td>16.5</td>
<td>33.3</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

(a) Without graphing, show that the seven given data values in the table are quadratic.
(b) Without finding an equation, fill in the missing values.
(c) Find a quadratic equation for the data.

1.—2.4.—ANSWER: (a) See diagram below. Because the second differences are constant, the data are (perfectly) quadratic; (b) 56.1, 84.9; (c) The equation is $y = 0.75x^2 + 8.1x + 22.5$.

The table shows the weekly profit from selling a certain type of rain gear at a large camping supply store.

<table>
<thead>
<tr>
<th>Number of rain gear sold</th>
<th>4</th>
<th>16</th>
<th>28</th>
<th>40</th>
<th>52</th>
<th>64</th>
<th>76</th>
</tr>
</thead>
<tbody>
<tr>
<td>Profit (dollars)</td>
<td>95</td>
<td>425</td>
<td>735</td>
<td>1026</td>
<td>1300</td>
<td>1554</td>
<td>1791</td>
</tr>
</tbody>
</table>

(a) Examine a scatter plot of the data, and find the second differences for the data in the table. What information do you obtain from each of these about what equation to use for the data?
(b) Find a quadratic model for the data.
(c) Find the profit when none of the rain gear and when 75 of the rain gear are sold.
(d) What use is the answer for the profit when no rain gear are sold?

1.—2.4.—ANSWER: (a) The scatter plot has a slight downward concavity. The second differences are -20, -19, -17, -20, and -17. These values are close to being constant, so a quadratic equation will probably fit the data fairly well. (b) When $t$ of the rain gear are sold, the profit is $P(t) = -0.0644t^2 + 28.6968t - 18.3042$ dollars; (c) Approximately -$18; about $1772; (d) The fixed costs for the sale of this type of rain gear are about $18.

The table shows the monthly profit of an amusement park for various ticket prices.

<table>
<thead>
<tr>
<th>Ticket price (dollars)</th>
<th>20</th>
<th>22</th>
<th>24</th>
<th>26</th>
<th>28</th>
<th>30</th>
</tr>
</thead>
<tbody>
<tr>
<td>Profit (millions of dollars)</td>
<td>14.4</td>
<td>19.4</td>
<td>23.1</td>
<td>25.2</td>
<td>26.0</td>
<td>25.2</td>
</tr>
</tbody>
</table>

(a) Is a quadratic model appropriate for the data? Explain.
(b) Find a quadratic model to fit the data.
(c) At what ticket price will the amusement park begin to post a net loss (i.e., a negative profit)?
(d) Use the function you found in part b to estimate the price that should be charged for a ticket to receive the highest profit.

2.—2.4.—ANSWER: (a) Yes, a scatter plot of the data shows a concave down shape and a definite maximum; (b) $P(t) = -0.1813t^2 + 10.1468t - 116.6571$ million dollars when tickets are priced at $t$ dollars each; (c) Profit begins to be negative when tickets are priced at about $40 each. (d) About $27.90
The table shows the value of one U.S. dollar in Italian lira for selected years from 1970 through 1989. (Source: International Monetary Fund)

<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>Value of $1 (in lira)</td>
<td>623</td>
<td>653</td>
<td>856</td>
<td>1302</td>
<td>1372</td>
</tr>
</tbody>
</table>

(a) Should the data be modeled by a linear function or a quadratic function? Explain.
(b) Find an appropriate model for the data. How well does the function fit the data?
(c) According to the model, how much would 1 U.S. dollar have been worth in Italian lira in 1988, 1989, 1991, and 1995? The actual values of $1 were 1241 lira in 1991 and 1629 lira in 1995. How do the model estimates compare to the actual values?
(d) Do you think this model would make reliable predictions in future years? Explain.

2.4 ANSWER: (a) A scatter plot of the data shows an upward concavity. The data would be better represented by a quadratic function; (b) \( P(x) = 1.973x^2 + 2.627x + 614.223 \) lira \( x \) years after 1970. The model appears to fit the data well. (c) \( P(18) \approx 1300.8 \) lira, \( P(19) \approx 1376.5 \) lira, \( P(21) \approx 1539.6 \) lira, and \( P(25) \approx 1913.1 \) lira. The model estimates very accurately for 1988 and 1989 but overestimates drastically for 1991 and 1995. (d) No, as the results of part c show, extrapolations can be quite inaccurate. There are many factors that may impact results outside the data interval. Results from extrapolations should be considered with extreme care.

The table shows the annual net income in millions of dollars for NIKE, Inc. between 1987 and 1993. (Source: Hoover's Online Guide)

<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>Net income (millions of dollars)</td>
<td>36</td>
<td>102</td>
<td>167</td>
<td>243</td>
<td>287</td>
<td>329</td>
<td>365</td>
</tr>
</tbody>
</table>

(a) Find the second differences in the output data. What do these values tell you about using a quadratic function to model the data in the table?
(b) Examine a scatter plot of the data, and find a quadratic model to fit the data.
(c) Graph the model on the scatter plot. Is the model a good fit?
(d) Use the result of part b to predict NIKE's 1994 net income.
(e) NIKE's actual 1994 net income was $299 million. Compare this to your estimate in part c and comment.

2.4 ANSWER: (a) The second differences are -1, 11, -32, -2, -6. These values are not at all constant. Thus, the differences do not give us any information about which model to use for the data. (b) \( N(x) = -3.9167x^2 + 79.2500x + 31.5952 \) million dollars \( x \) years after 1987; (c) See graph; the model seems to fit the data well; (d) \( N(7) \approx 394 \) million; (e) The estimated net income is $95 million higher than the actual 1994 net income. Extrapolation, even when it is done right outside of the collected data, should always be viewed with caution.
The number of households subscribing to cable television in the United States is given in the table. (Source: Statistical Abstract, 1998)

(a) Using an input of the number of years from 1960, find a quadratic equation to fit the aligned data. Comment on the fit of the function to the data.

(b) Using an input of the number of years from 1960, fit an exponential equation to the aligned data. Comment on the fit of the function to the data.

(c) Use each model to predict the number of subscribers to cable TV in 1997.

(d) The actual number of subscribers in 1997 was 64,050 households. Which of the answers to part c comes closer to this value?

(e) Does this result agree with your observations in part b?

1—2.4—ANSWER: (a) \( Q(x) = 53.173x^2 - 113.958x + 259.375 \) thousand households \( x \) years after 1960. The model fits the data fairly well. (b) \( E(x) = 892.447(1.14306^x) \) thousand households \( x \) years after 1960. The model is not a good fit as the quadratic equation because it does not seem to follow the shape of the data after 1970. (c) In 1997, \( Q(37) \approx 68,836 \) households and \( E(37) \approx 125,658 \) households; (d) Even though both functions overestimate, the estimate using quadratic equation is much closer the actual 1997 value. (e) Yes, this agrees with the graphical view of the better-fitting function.

The scatter plot in the figure projects the ratio of the working-age population to the elderly. (Source: Newsweek, December 1999)

(a) Is the function defined by the scatter plot a discrete or continuous function?

(b) Should the data in this figure be modeled by a quadratic function? Explain.

(c) Would the coefficient of the squared term in a quadratic function fit to these data be positive or negative?

1—2.4—ANSWER: (a) All scatter plots are discrete. (b) Because these data are concave down and decreasing with no evidence of a limiting value, it appears that a quadratic function would fit. (c) Negative
The table shows the United States budget receipts from individual income taxes for each fiscal year between 1990 and 2000. (Source: U.S. Department of the Treasury)

<table>
<thead>
<tr>
<th>Year</th>
<th>Individual taxes (billions of dollars)</th>
<th>Year</th>
<th>Individual taxes (billions of dollars)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1990</td>
<td>466.9</td>
<td>1996</td>
<td>656.4</td>
</tr>
<tr>
<td>1991</td>
<td>467.8</td>
<td>1997</td>
<td>737.5</td>
</tr>
<tr>
<td>1992</td>
<td>476.0</td>
<td>1998</td>
<td>828.6</td>
</tr>
<tr>
<td>1993</td>
<td>509.7</td>
<td>1999</td>
<td>879.5</td>
</tr>
<tr>
<td>1994</td>
<td>543.1</td>
<td>2000</td>
<td>1004.5</td>
</tr>
<tr>
<td>1995</td>
<td>590.2</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

(a) Examine a scatter plot of the data, and describe the concavity of the graph.
(b) Find a quadratic model for the data.
(c) Can the continuous quadratic function in part b be used without restriction or must it be discretely interpreted?
(d) Graph the model on the scatter plot of the data. Comment on the fit.
(e) Use the model to estimate the budget receipts from individual income taxes in 2001.

2—2.4—ANSWER: (a) The graph is concave up between 1990 and 2000. (b) A quadratic model is $T(x) = 5.4423x^2 - 0.2276x + 461.5846$ billion dollars $x$ years after 1990; (c) The function $T$ must be discretely interpreted because income taxes from individuals are tabulated only once each year. (d) See graph below. The model seems to fit the data well. (e) $T(11) = \$1117.6$ billion dollars

![Graph of individual income taxes](image-url)
The yearly consumer spending per person for television subscription video services is given in the table. (Source: Statistical Abstract, 1998)

<table>
<thead>
<tr>
<th>Year</th>
<th>Consumer spending (dollars per person per year)</th>
<th>Year</th>
<th>Consumer spending (dollars per person per year)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1994</td>
<td>110.89</td>
<td>1998</td>
<td>168.75</td>
</tr>
<tr>
<td>1995</td>
<td>125.54</td>
<td>1999*</td>
<td>178.23</td>
</tr>
<tr>
<td>1996</td>
<td>140.37</td>
<td>2000*</td>
<td>187.60</td>
</tr>
<tr>
<td>1997</td>
<td>155.70</td>
<td>2001*</td>
<td>196.62</td>
</tr>
</tbody>
</table>

*Projected

(a) Find a quadratic model that gives the consumer spending as a function of the number of years from 1990.

(b) Find a quadratic model that gives the consumer spending as a function of the number of years from 1994.

(c) When does the per capita yearly consumer spending for television subscription video services first exceed $215?

(d) What assumption must be made for the answer to part (c) to be considered valid?

2---2.4—ANSWER: (a) \( G(x) = -0.6687x^2 + 22.3761x + 31.2664 \) dollars per person per year, where \( x \) is the number of years after 1990; (b) \( P(t) = -0.6687t^2 + 17.0265t + 110.0717 \) dollars per person per year \( t \) years after 1994; (c) 2005; (d) We assume that the pattern indicated by the data continues through the year 2005.


<table>
<thead>
<tr>
<th></th>
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<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>Sales (millions of dollars)</td>
<td>2</td>
<td>17</td>
<td>71</td>
<td>276</td>
<td>627</td>
<td>1107</td>
<td>1732</td>
<td>2701</td>
<td>3676</td>
<td>5035</td>
</tr>
</tbody>
</table>

(a) Should the data be modeled by a linear or a quadratic model? Explain.

(b) Find an appropriate model for the data. How well does the model fit the data?

(c) Use the model to estimate Gateway 2000’s annual sales in 1997.

(d) According to the model, when will Gateway 2000’s annual sales reach $20,520 million?

1---2.4—ANSWER: (a) A scatter plot of the data shows an upward concavity. The data would be better represented by a quadratic model; (b) \( S(x) = 82.826x^2 - 206.363x + 92.091 \) million dollars \( x \) years after 1987. The model appears to fit the data well. (c) \( S(10) \approx 6310.8 \) million; (d) When \( x \approx 17 \), or about 2004.
The table shows the U.S. public debt per capita from 1988 through 1995. (Source: U.S. Department of the Treasury)

<table>
<thead>
<tr>
<th></th>
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<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>Debt</td>
<td>10,534</td>
<td>11,545</td>
<td>13,000</td>
<td>14,436</td>
<td>15,846</td>
<td>17,105</td>
<td>18,025</td>
<td>18,930</td>
</tr>
<tr>
<td>(dollars per person)</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

(a) Find a quadratic model for the data.
(b) Estimate the public debt per capita in 1996.
(c) According to your model, will there ever be a year in which the per capita debt is more than $24,000? more than $27,000?

---

1—2.4—ANSWER: (a) \( D(x) = -35.161x^2 + 1494.899x + 10,310.792 \) dollars per person \( x \) years after 1988; (b) \( D(8) = 20,020 \) per person; (c) The model peaks at about $26,200 dollars per person. Therefore, according to the model, the per capita debt will rise above $24,000 but will never reach $27,000.
The income received by persons from all sources minus their personal contributions for social insurance. The table lists the personal income, in billions of dollars, of persons living in the United States for the indicated years. (Source: Statistical Abstract, 1998)

<table>
<thead>
<tr>
<th></th>
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<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>Income (billions of dollars)</td>
<td>4804.2</td>
<td>4981.6</td>
<td>5277.2</td>
<td>5519.2</td>
<td>5791.8</td>
<td>6150.8</td>
<td>6495.2</td>
<td>6873.9</td>
</tr>
</tbody>
</table>

(a) Using an input equal to the number of years after 1990, find a quadratic function that models these data.
(b) Use your model to estimate when the personal income was $5,500,000,000.
(c) If the pattern indicated by the model remains valid, when will the personal income be at least double its 1990 value?

1—2.4—ANSWER: (a) \( f(x) = 14.193x^2 + 197.661x + 4796.538 \) billion dollars \( x \) years after 1990; (b) Solve \( f(x) = 5500 \) to find \( x \approx 2.9 \), so 1993; (c) Solve \( f(x) = 2(4802.4) \) to find \( x \approx 12.7 \), so the answer is 2003.

63.

(a) Describe the curvature of the graph in the figure by identifying the portions of the horizontal axis over which the graph is concave up or concave down.
(b) Identify the approximate location of the inflection point.
(c) Over which portions of the horizontal axis is the graph increasing or decreasing?
(d) What is the name of the function that is graphed in the figure?

1—2.4—ANSWER: (a) Concave up from approximately -2 to about 10; concave down from about 10 to 22; (b) Inflection point at approximately (10, 1350); (c) Decreasing from -2 to about 5 and about 15 to 22; increasing from about 5 to 15; (d) Cubic function

64.

(a) Describe the curvature of the graph in the figure by identifying the portions of the horizontal axis over which the curve is concave up or concave down.
(b) Identify the approximate location of the inflection point.
(c) Over which portions of the horizontal axis is the curve decreasing?
(d) What is the name of the function that is graphed in the figure?

1—2.4—ANSWER: (a) Concave down from approximately 0 to about 5; concave up from about 5 to almost 10; (b) Inflection point at approximately (5, 40); (c) Decreasing from about 3 to approximately 6.5; (d) Cubic function
5. (a) Describe the curvature of the graph in the figure by identifying the portions of the horizontal axis over which the curve is concave up or concave down.
(b) Identify the approximate location of the inflection point.
(c) Over which portions of the horizontal axis is the curve increasing or decreasing?

1—2.4—ANSWER: (a) Concave down from 0 to about 50; concave up from about 50 to 100; (b) Inflection point at approximately (50, 5); (c) Always increasing

66. Refer to the table below.

<table>
<thead>
<tr>
<th>x</th>
<th>0</th>
<th>2</th>
<th>4</th>
<th>6</th>
<th>8</th>
<th>10</th>
<th>12</th>
</tr>
</thead>
<tbody>
<tr>
<td>y</td>
<td>27</td>
<td>57</td>
<td>52</td>
<td>29</td>
<td>8</td>
<td>44</td>
<td>150</td>
</tr>
</tbody>
</table>

(a) Make a scatter plot of the data.
(b) What type of model would be appropriate for these data?
(c) Find an appropriate equation for the data.

1—2.4—ANSWER: (a) See graph below; (b) Cubic; (c) A cubic function to fit these data is

\[ y = 0.625x^3 - 9.497x^2 + 34.304x + 25.452. \]

67. (a) Describe the curvature of the graph in the figure by identifying the portions of the horizontal axis over which the curve is concave up or concave down.
(b) Identify the approximate location of the inflection point.
(c) Over which portions of the horizontal axis is the curve increasing or decreasing?

1—2.4—ANSWER: (a) Concave down from 0 to about 11; concave up from about 11 to 16.5; (b) Inflection point at approximately (11, 90); (c) Increasing from 0 to about 5.5; decreasing from about 5.5 to 16.5
Refer to the table below.

<table>
<thead>
<tr>
<th>x</th>
<th>0</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
<th>7</th>
<th>8</th>
<th>9</th>
<th>10</th>
</tr>
</thead>
<tbody>
<tr>
<td>y</td>
<td>73</td>
<td>61</td>
<td>55</td>
<td>53</td>
<td>56</td>
<td>60</td>
<td>66</td>
<td>72</td>
<td>76</td>
<td>79</td>
<td>77</td>
</tr>
</tbody>
</table>

(a) Draw a scatter plot of these data.
(b) Decide whether a quadratic or cubic model would be appropriate.
(c) Find an appropriate equation for the data.

1—2.4—ANSWER: (a) See graph below; (b) A cubic model would be appropriate because there is a change in concavity and no limiting values are indicated. A quadratic model is not appropriate because there is an inflection point. (c) \(y = -0.199x^3 + 3.588x^2 - 15.521x + 73.070\)

The table shows the number of Coca-Cola Company employees from 1987 through 1996.
(Source: Hoover's Online Guide)

<table>
<thead>
<tr>
<th></th>
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<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>Employees (thousands)</td>
<td>17.40</td>
<td>18.70</td>
<td>20.96</td>
<td>24.00</td>
<td>28.90</td>
<td>31.30</td>
<td>34.00</td>
<td>33.00</td>
<td>32.00</td>
<td>26.00</td>
</tr>
</tbody>
</table>

(a) Examine a scatter plot of the data. Find a cubic model to fit the data.
(b) Graph the model on a scatter plot of the data. Discuss how well the model fits the data.
(c) Estimate the input location of the inflection point.
(d) Use the model to estimate the number of Coca-Cola employees in 1997.
(e) What trend does the model indicate beyond 1996? Do you believe the trend is realistic? Explain.

2—2.4—ANSWER: (a) \(E(x) = -0.114x^3 + 1.129x^2 + (4.817 \times 10^{-4})x + 17.453\) thousand employees \(x\) years after 1987; (b) See graph below. The model seems to fit the data well.
(c) The input location of the inflection point seems to be just to the right of 1990. (d) \(E(10) \approx 16.7\) thousand employees; (e) The function shows a rapid decline after 1996. No, the trend is not realistic because it indicates that Coca-Cola would have no employees by about 1998.
The number of students enrolled each fall in higher education in United States schools is given in the table. (Source: Statistical Abstract, 1998)

<table>
<thead>
<tr>
<th></th>
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<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>Higher education enrollment each fall (millions of students)</td>
<td>12.2</td>
<td>13.8</td>
<td>14.3</td>
<td>14.9</td>
<td>15.5</td>
<td>16.1</td>
</tr>
</tbody>
</table>

(a) Examine a scatter plot and explain why a cubic model is appropriate for these data.
(b) Find a cubic function for these data using the year as input.
(c) Find a cubic function for these data using the number of years past 1980 as input.
(d) Find a cubic function for these data using the number of years past 1985 as input.
(e) Find and interpret the point on the function in part c with input equal to 24.
(f) Find and interpret the point on the function in part d with input equal to 19.

1—2.4—ANSWER: (a) The data indicate an inflection point no obvious limiting values.
(b) \( E1(t) = (6.247 \times 10^{-6})t^3 - 3.745t^2 + 7482.226t - 4,983,404.379 \) million students enrolled in higher education each fall in year \( t \); (c) \( E2(x) = (6.247 \times 10^{-4})x^3 - 0.034x^2 + 0.693x + 9.528 \) million students enrolled in higher education each fall \( x \) years after 1980; (d) \( E3(s) = (6.247 \times 10^{-4})s^3 - 0.024s^2 + 0.402s + 12,227 \) million students enrolled in higher education each fall \( s \) years after 1985; (e) \( E2(24) \approx 15.3 \) million students. Approximately 15.3 million students were enrolled in higher education in the U.S. in 2004. (f) \( E3(19) \approx 15.3 \) million students. The U.S. higher education enrollment in 2004 was about 15.3 million students.

Decide which function(s) are candidates to fit to the data shown in the scatter plot. Choose from linear, quadratic, cubic, exponential, log, and logistic models. Explain why the candidate(s) is/are appropriate.

1—2.6—ANSWER: Because the data are concave up, display a minimum point, and show no changes in concavity, a quadratic function is the only appropriate choice.
12. Decide which function(s) are candidates to fit to the data shown in the scatter plot. Choose from linear, quadratic, cubic, exponential, log, and logistic models. Explain why the candidate(s) is/are appropriate.

1—2.6—ANSWER: Because the data display a change in concavity (an inflection point) but do not exhibit limiting values, a cubic function is the only appropriate choice.

13. Decide which function(s) are candidates to fit to the data shown in the scatter plot. Choose from linear, quadratic, cubic, exponential, log, and logistic models. Explain why the candidate(s) is/are appropriate.

1—2.6—ANSWER: Because the data are concave up, increasing, and do not display an obvious minimum, either a quadratic or an exponential function is an appropriate candidate.

14. Decide which function(s) are candidates to fit to the data shown in the scatter plot. Choose from linear, quadratic, cubic, exponential, log, and logistic models. Explain why the candidate(s) is/are appropriate.

1—2.6—ANSWER: The data appear to be essentially linear. Any observed concavity is probably not obvious enough to warrant the use of a more complex model.
75. Decide which function(s) are candidates to fit to the data shown in the scatter plot. Choose from linear, quadratic, cubic, exponential, log, and logistic models. Explain why the candidate(s) is/are appropriate.

1—2.6—ANSWER: Because the data are concave up, decreasing, and seem to be approaching a limiting value on the right, an exponential or log model could be considered.

76. Decide which function(s) are candidates to fit to the data shown in the scatter plot. Choose from linear, quadratic, cubic, exponential, log, and logistic models. Explain why the candidate(s) is/are appropriate.

1—2.6—ANSWER: Because the data are concave down, and decreasing, a quadratic function is a candidate. The data can be observed as approaching a limiting value as \( x \) increases, so a log model is also a candidate.

77. The table shows data recorded on December 31 of the indicated years for the number of U.S. air carrier accidents, all services. (Source: Statistical Abstract, 1998)

<table>
<thead>
<tr>
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<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>Accidents</td>
<td>37</td>
<td>19</td>
<td>21</td>
<td>24</td>
<td>36</td>
<td>38</td>
<td>49</td>
</tr>
</tbody>
</table>

(a) Examine a scatter plot of the data and discuss whether each of the following functions is appropriate as a candidate to fit the data: linear, quadratic, cubic, exponential, log, and/or logistic.

(b) Write the best-fit model. (Be certain to give an equation, define the variables you use, and include units of measure for all variables.)

(c) Without finding the value, determine if an estimate from the model of the number of accidents in 1980 would be more or less than the data value in the table. Explain your reasoning.

2—2.6—ANSWER: (a) The data show curvature, so a linear function is not appropriate. Because the data are concave up and exhibit a minimum point, a quadratic model is a candidate. There is no inflection point, so neither a cubic nor a logistic model should be considered. Because of the minimum point, neither an exponential or log model would be a good choice. A quadratic function is the only choice. (b) \( A(x) = 0.184x^2 - 3.599x + 35.836 \) accidents where \( x \) is the number of years past 1975. (c) When \( x = 5 \), the parabola goes above the 1980 data point. Thus, the model estimate would be larger than 19 accidents.
78. The table shows the percent of households in the United States with computers from 1992 through 1996. Examine the data set, and determine the best model. Give the equation, define the variables you use, include units of measure, and explain why you chose that model. (Source: Electronics Industry Association)

<table>
<thead>
<tr>
<th>Year</th>
<th>Households with computers (percent)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1992</td>
<td>27</td>
</tr>
<tr>
<td>1993</td>
<td>30</td>
</tr>
<tr>
<td>1994</td>
<td>33</td>
</tr>
<tr>
<td>1995</td>
<td>37</td>
</tr>
<tr>
<td>1996</td>
<td>40</td>
</tr>
</tbody>
</table>

1—2.6—ANSWER: The first differences are 3, 3, 4, and 3. Because these are close to constant, we suspect that the data are close to linear. A scatter plot confirms this observation. The best model is $P(x) = 3.3x + 20.8$ percent $x$ years after 1992.

79. The table shows the annual net income of a small company from 1992 through 1996.

<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>Net income (thousands of dollars)</td>
<td>10,230</td>
<td>12,070</td>
<td>13,410</td>
<td>14,270</td>
<td>14,640</td>
</tr>
</tbody>
</table>

Examine the data set, and determine the best model. Give the equation, define the variables you use, include units of measure, and explain why you chose that model.

1—2.6—ANSWER: Because a scatter plot shows that the data are concave down (with no changes in concavity) an do not appear to be growing slowly, the best choice is a quadratic function. The best model is $N(x) = -244.286x^2 + 2079.143x + 10,231.4291$ thousand dollars $x$ years after 1992.

80. The table shows the cumulative number of paper grocery bags used at a grocery store over a period of several weeks.

<table>
<thead>
<tr>
<th>Week</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
<th>7</th>
<th>8</th>
</tr>
</thead>
<tbody>
<tr>
<td>Cumulative number of paper bags used</td>
<td>1253</td>
<td>2360</td>
<td>3462</td>
<td>4560</td>
<td>5665</td>
<td>6765</td>
<td>7868</td>
<td>8970</td>
</tr>
</tbody>
</table>

Examine the data set, and determine the best model. Give the equation, define the variables you use, include units of measure, and explain why you chose that model.

1—2.6—ANSWER: A scatter plot shows the data to be essentially linear. The best model is $B(x) = 1102.060x + 153.607$ paper bags used after $x$ weeks.

81. The table shows the emissions of lead into the air in thousands of short tons in the United States from 1986 through 1992. (Source: U.S. Environmental Protection Agency)

<table>
<thead>
<tr>
<th></th>
<th></th>
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<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>Lead emissions (thousand short tons)</td>
<td>7.3</td>
<td>6.9</td>
<td>6.5</td>
<td>6.0</td>
<td>5.7</td>
<td>5.3</td>
<td>4.9</td>
</tr>
</tbody>
</table>

Examine the data set, and determine the best model. Give the equation, define the variables you use, include units of measure, and explain why you chose that model.

1—2.6—ANSWER: A scatter plot shows the data to be essentially linear. The best model is $L(x) = -0.400x + 7.286$ thousand short tons $x$ years after 1986.