Change, Percentage Change, and Average Rates of Change

1. Rewrite the following sentence to express how rapidly, on average, the quantity changed over the given interval: Mortgage rates have fallen 2 percentage points during the past 3 years.

1—3.1—ANSWER: Mortgage rates have fallen an average of 0.67 percentage point per year over the past 3 years.

2. Rewrite the following sentence to express how rapidly, on average, the quantity changed over the given interval: The child grew 10 inches over the past 4 years.

1—3.1—ANSWER: The child grew an average of 2.5 inches per year over the past 4 years.

3. Rewrite the following sentence to express how rapidly, on average, the quantity changed over the given interval: Over 6 months, the board shrank 0.3 inches in length.

1—3.1—ANSWER: The board shrank in length by an average of 0.05 inch per month during the 6-month period.

4. Rewrite the following sentence to express how rapidly, on average, the quantity changed over the given interval: The traffic safety official counted 818 cars in 4 hours.

1—3.1—ANSWER: The traffic safety official counted an average of 204.5 cars per hour during the 4-hour period.

5. Rewrite the following sentence to express how rapidly, on average, the quantity changed over the given interval: The estimated per-gallon increase in price for cleaner-burning reformulated gasoline increased from 5.4 cents in 1995 to 7.4 cents in 2000. (Source: American Petroleum Institute)

1—3.1—ANSWER: The estimated per-gallon increase in price for cleaner-burning reformulated gasoline increased an average of 0.4 cent per year between 1995 and 2000.


1—3.1—ANSWER: Average rate of change = −116.933 thousand automobiles per year
7. The number of cars produced by Saturn Corporation increased from 212,122 in 1992 to 301,540 in 1995. Calculate the change, average rate of change, and percentage change in the number of Saturns produced from 1992 through 1995. (Source: American Automobile Manufacturers Association)

1—3.1—ANSWER: Change = 89,418 cars; average rate of change = 29,806 cars per year; percentage change ≈ 42.2%

8. In 1996 there were 68,850 commissioned officers on active duty in the United States Army. That was a decrease from 85,953 commissioned officers on active duty in 1992. Calculate each of the following for the number of commissioned officers on active duty in the United States Army from 1992 through 1996. (Source: U.S. Department of Defense) Write a sentence interpreting each description of change that you calculate.
   (a) Change
   (b) Percentage change
   (c) Average rate of change

1—3.1—ANSWER: (a) Change = -17,103 commissioned officers; Between 1992 and 1996 the number of commissioned officers in the U.S. Army fell by 17,103. (b) Percentage change ≈ -19.9%; The number of commissioned officers in the U.S. Army fell by about 19.9% between 1992 and 1996. (c) Average rate of change = -4275.75 commissioned officers per year; The number of commissioned officers in the U.S. Army fell at an average rate of about 4276 commissioned officers per year.

9. In 1992, 14.8% of United States citizens were living in poverty. In 1995, only 13.8% of United States citizens were living in poverty. (Source: Bureau of the Census) Calculate each of the following for the percentage of citizens living in poverty from 1992 through 1995. Write a sentence interpreting each description of change.
   (a) Change
   (b) Percentage change
   (c) Average rate of change

1—3.1—ANSWER: (a) Change = -1 percentage point; Between 1992 and 1995 the percentage of U.S. citizens living in poverty declined by 1 percentage point. (b) Percentage change ≈ -6.76%; The percentage of U.S. citizens living in poverty declined by about 6.76% between 1992 and 1995. (c) Average rate of change ≈ -0.33 percentage point per year; The percentage of citizens living in poverty declined at an average rate of about 0.33 percentage point per year.

   (a) Change
   (b) Percentage change
   (c) Average rate of change

1—3.1—ANSWER: (a) Change = 1300 officers; The Houston police force size increased by 1300 officers between 1991 and 1995. (b) Percentage change ≈ 33.3%; The size of the Houston police force increased by about 33.3% between 1991 and 1995. (c) Average rate of change = 325 officers per year; Houston's police force increased by an average of 325 officers per year between 1991 and 1995.
1. Between 1980 and 1997 the total U.S. farm acreage decreased from 1 billion to 968 million acres. (Source: United States Department of Agriculture) Calculate each of the following for the total U.S. farm acreage from 1980 through 1997. Write a sentence interpreting each description of change. (a) Change  (b) Percentage change  (c) Average rate of change

1—3.1—ANSWER: (a) Change = -32 million acres; Between 1980 and 1997 Total U.S. farm acreage decreased by 32 million acres. (b) Percentage change = -3.2%; Total U.S. farm acreage decreased by 3.2% from 1980 through 1997. (c) Average rate of change ≈ -1.9 million acres per year; Total U.S. farm acreage decreased an average rate of about 1.9 million acres per year between 1980 and 1997.

2. The graph shows the total number of crimes in millions reported in the United States from 1992 through 1999. (Source: Based on data from FBI, Uniform Crime Reports, 1999) 
(a) Sketch a secant line connecting the beginning and ending points of the graph. Find the slope of this line.
(b) Write a sentence interpreting the slope in the context of the number of crimes reported.
(c) Write a sentence summarizing how the actual number of crimes reported changed from 1992 through 1999. How well does your answer to part b describe the change in the number of crimes as shown in the graph?

2—3.1—ANSWER: (a) Slope of secant line ≈ -0.39 million crimes per year; (b) In the 7-year period from 1992 through 1999, the number of crimes reported dropped an average of 0.39 million crimes per year. (c) The actual number of crimes reported decreased slowly from 1992 through 1995 and then decreased more rapidly each year through 1999. The average rate of change describes the year-to-year decrease, but it does not show the different rates of decrease.

3. The number of Internet brokerage accounts between 1996 and 2001 is as given in the table. (Source: Time, May 31, 1999)
(a) Find the slope of the line joining the data points for 1996 and 2000. (Include units with your answer.)
(b) How quickly was the number of accounts changing between 1996 and 2000?
(c) Find the average rate of change in the number of accounts between 1997 and 1999.
(d) Find the percentage change in the number of accounts between 1998 and 2000.
(e) Interpret your answer to part d.

2—3.1—ANSWER: (a) Slope of line = 3.125 million accounts per year; (b) Increasing by 3.125 million accounts per year; (c) Average rate of change ≈ 3.2 million accounts per year; (d) Percentage change ≈ 153.5%; (e) Between 1998 and 2001 the number of Internet brokerage accounts increased by about 153.5%.
14. The table shows the number of people who participated in a newly launched free HIV-testing program.

<table>
<thead>
<tr>
<th>Month</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>100</td>
<td>168</td>
<td>261</td>
<td>409</td>
<td>592</td>
<td>817</td>
</tr>
</tbody>
</table>

(a) Use the data to find the average rate of change in the number of participants from month 1 through month 6.
(b) Fit a model to the data.
(c) Use the model to find the average rate of change in the number of participants from month 1 through month 6.
(d) Compare your results from part c to the result of part a. Which is more accurate? Why?

2—3.1—ANSWER: (a) Average rate of change = 143.4 participants per month; (b) The model is \( P(x) = 20.446x^2 - 0.125x + 81.5 \) participants after \( x \) months; (c) Average rate of change = 143 participants per month; (d) The answer obtained from the data is more accurate than that from the model because the model is only an approximation of what took place whereas the data give the actual numbers of participants.

The table shows the monthly profit of an amusement park for various ticket prices.

<table>
<thead>
<tr>
<th>Ticket price (dollars)</th>
<th>20</th>
<th>22</th>
<th>24</th>
<th>26</th>
<th>28</th>
<th>30</th>
</tr>
</thead>
<tbody>
<tr>
<td>Profit (millions of dollars)</td>
<td>14.4</td>
<td>19.4</td>
<td>23.1</td>
<td>25.2</td>
<td>26.0</td>
<td>25.2</td>
</tr>
</tbody>
</table>

(a) Use the data to find the average rate of change of profit when the ticket price rises from $20 to $26.
(b) Fit a model to the data.
(c) Estimate the average rate of change of profit when the ticket price rises from $20 to $23.
(d) How rapidly is profit changing when the ticket price rises from $23 to $29?

2—3.1—ANSWER: (a) Average rate of change = $1.8 million in profit per dollar of ticket price; (b) \( P(t) = -0.181t^2 + 10.147t - 116.057 \) million dollars in profit when tickets are priced at \( t \) dollars each; (c) Average rate of change \( \approx \) $2.353 million in profit per dollar of ticket price; (d) Average rate of change \( \approx \) $0.722 million in profit per dollar of ticket price.

The table shows the amount of energy imported by the United States for selected years from 1960 through 1994. (Source: U.S. Department of Energy)

<table>
<thead>
<tr>
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<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>Energy (quadrillion Btu)</td>
<td>4.23</td>
<td>8.39</td>
<td>18.99</td>
<td>19.66</td>
<td>21.54</td>
<td>22.71</td>
</tr>
</tbody>
</table>

(a) Use the data to find the average rate of change of imported energy from 1960 through 1992.
(b) Find an exponential model for the data.
(c) Estimate the average rate of change of imported energy from 1965 through 1971.
(d) How rapidly were energy imports changing from 1975 through 1988?

2—3.1—ANSWER: (a) Average rate of change \( \approx \) 0.482 quadrillion Btu per year; (b) \( E(x) = 4.623(1.047855^x) \) quadrillion Btu \( x \) years after 1960; (c) Average rate of change \( \approx \) 0.315 quadrillion Btu per year; (d) Average rate of change \( \approx \) 0.600 quadrillion Btu per year
7. The number of bank failures in the United States from 1988 through 1995 can be modeled by
\[ F(x) = \frac{234.779}{1 + 0.061e^{0.889180x}} \] failures x years after 1988. (Source: Based on data from the Federal Deposit Insurance Corporation)
(a) How much did the number of bank failures change from 1988 through 1992?
(b) What was the average rate of change in the number of bank failures from 1988 through 1992?
(c) How quickly was the number of bank failures changing from 1988 through 1990?
(d) How rapidly was the number of bank failures changing from 1993 through 1995?
(e) Interpret your result from part d.

2—3.1—ANSWER: (a) Change \( \approx -146.5 \) failures; (b) Average rate of change \( \approx -36.6 \) failures per year; (c) Approximately -24.4 per year; (d) Approximately -15.2 per year; (e) The number of bank failures was decreasing by about 15.2 failures per year over the 2-year period from 1993 through 1995.

8. The table shows the number of people (in thousands) employed by PepsiCo, Inc., from 1987 through 1996. (Source: Hoover's Company Profiles, 1997)

<table>
<thead>
<tr>
<th></th>
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<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>Employees (thousands)</td>
<td>225</td>
<td>235</td>
<td>266</td>
<td>308</td>
<td>338</td>
<td>372</td>
<td>423</td>
<td>471</td>
<td>480</td>
<td>486</td>
</tr>
</tbody>
</table>

(a) Find the first differences in the number of employees, and record these as average rates of change.
(b) When was the number of employees growing most rapidly? How rapidly (on average) was the number of employees growing at that time?
(c) When was the number of employees growing least rapidly? How rapidly (on average) was the number of employees growing at that time?

2—3.1—ANSWER: (a) Average rates of change: 10, 31, 42, 30, 34, 51, 48, 9, and 6 thousand employees per year; (b) Between 1992 and 1993; 51 thousand employees per year; (c) Between 1995 and 1996; 6 thousand employees per year.

9. The table shows the outdoor air temperature in degrees Fahrenheit between 4 a.m. and 8 a.m. for a small town in Nebraska in late August.

<table>
<thead>
<tr>
<th>Time</th>
<th>4:00</th>
<th>4:30</th>
<th>5:00</th>
<th>5:30</th>
<th>6:00</th>
<th>6:30</th>
<th>7:00</th>
<th>7:30</th>
<th>8:00</th>
</tr>
</thead>
<tbody>
<tr>
<td>Temperature (°F)</td>
<td>56.0</td>
<td>54.5</td>
<td>53.3</td>
<td>52.5</td>
<td>52.1</td>
<td>52</td>
<td>52.3</td>
<td>53</td>
<td>54</td>
</tr>
</tbody>
</table>

(a) Find the first differences in the temperatures, and convert these to average rates of change.
(b) When was the temperature increasing most rapidly? How rapidly (on average) was the temperature increasing at that time?
(c) When was the temperature decreasing least rapidly? How rapidly (on average) was the temperature decreasing at that time?

2—3.1—ANSWER: (a) Average rates of change: -3, -2.4, -1.6, -0.8, -0.2, 0.6, 1.4, and 2 degrees per hour; (b) Between 7:30 a.m. and 8:00 a.m.; increasing by 2 degrees per hour; (c) Between 6:00 a.m. and 6:30 a.m.; decreasing by 0.2 degrees per hour.
The number of pieces of priority mail delivered by the United States Postal Service from 1992 through 1996 can be modeled by $P(x) = 95.46x + 578.44$ million pieces $x$ years since 1992. (Source: Based on data from 1996 USPS Annual Report)

(a) Calculate the change and percentage change in priority mail delivery from 1992 through 1996.

(b) Interpret the change and percentage change calculated in part $a$.

(c) Calculate the average rate of change in priority mail delivery from 1992 through 1996.

$2—3.1$—ANSWER:  (a) Change = 381.84 million pieces; percentage change $\approx 66\%$; (b) The number of pieces of priority mail delivered in 1996 was 381.84 million more than the number in 1992. That change represents a 66% increase in priority mail delivery. (c) Between 1992 and 1996, priority mail delivery increased by an average of 95.46 million pieces per year.

The sales made by Gateway 2000, Inc., from 1987 through 1996 can be described by the model $S(x) = 82.826x^2 - 206.83x + 92.091$ million dollars $x$ years after 1987. (Source: Based on data from Hoover's Company Profiles)

(a) Calculate the change and percentage change in sales from 1987 through 1996.

(b) Interpret the change and percentage change calculated in part $a$.

(c) Calculate the average rate of change from 1987 through 1996.

$2—3.1$—ANSWER:  (a) Change = $4851.459$ million dollars; percentage change $\approx 5268\%$; (b) Gateway 2000's sales in 1996 were $4851.459$ million dollars more than the sales in 1987. That change represents a 5268% increase in sales; (c) Between 1987 and 1996, sales increased by an average of $539.051$ million dollars per year.

The number of people employed by the Gap, Inc., from 1988 through 1997 can be modeled by $E(x) = -0.041x^3 + 0.851x^2 + 1.236x + 16.577$ thousand employees $x$ years after 1988. (Source: Based on data from Hoover's Company Profiles)

(a) Calculate the change and percentage change in employees from 1989 through 1997.

(b) Interpret the change and percentage change calculated in part $a$.

(c) Calculate the average rate of change from 1988 through 1997.

$2—3.1$—ANSWER:  (a) Change = 50.166 thousand employees; percentage change $\approx 302.6\%$; (b) In 1997, there were 50.166 thousand more employees than in 1988. That change represents a 302.6% increase in employees. (c) Between 1988 and 1997, the number of Gap employees increased by an average of 5.574 thousand employees per year.

The U.S. public debt per capita from 1988 through 1995 can be described by the equation $D(x) = -35161x^2 + 1494.899x + 10,310.792$ dollars per person $x$ years after 1988. (Source: Based on data from the U.S. Department of the Treasury)

(a) Calculate the change and percentage change in per capita public debt from 1988 through 1995.

(b) Calculate the average rate of change in per capita public debt from 1988 through 1995.

$2—3.1$—ANSWER:  (a) Change = $8741.404$ per person; percentage change $\approx 84.8\%$; (b) Between 1988 and 1995, public debt increased by an average of $1248.772$ per person per year.
14. Explain the meaning of instantaneous rate of change.

1—3.2—ANSWER: An instantaneous rate of change at a point is the slope of the line tangent to the graph at that point.

Explain, using your own words, the principle of local linearity.

1—3.2—ANSWER: Answers may vary but should contain the information in this statement: If we zoom in close enough to a point on a smooth curve, the curve at that point looks like the line tangent to the curve at that point.

15. As a general rule, a tangent line to a smooth curve lies completely on one side of the curve near the point of tangency. Is this always the case? Explain.

2—3.2—ANSWER: No; if the point of tangency is the inflection point on the smooth curve, the tangent line will cut through the curve at the point of tangency. If the smooth curve is a line, the tangent line at any point of tangency will coincide with the original line.

16. Which of the lines drawn on the graph are tangent lines? Explain.

1—3.2—ANSWER: The line at A is tangent because it lies completely on one side of the curve and touches the curve at only one point near A. It is also tilted the way the curve is tilted at A. The line at B is not tangent to the curve because it cuts through the curve at B, which is not an inflection point.

27. Determine whether each line drawn on the graph is a tangent line. Explain.

1—3.2—ANSWER: The line at A is not a tangent line because it does not lie completely on one side of the curve and A is not an inflection point. The line at B is tangent because it lies completely on one side of the curve, is tilted the way the curve is tilted, and touches the curve at only one point, namely B.
28. Explain whether a line can be drawn tangent to the curve at point B.

2—3.2—ANSWER: No, because secant lines drawn with points on the right of B and on the left of B do not approach the same slope, there is no tangent line that can be drawn at B.

Refer to the graph.
(a) Is the graph concave up, concave down, or neither at A, B, C, D, and E?
(b) If a tangent line cannot be drawn at a particular point, explain why not. Should the tangent lines lie above or below the curve at each of the remaining points?

2—3.2—ANSWER: At point A, the graph is concave down and the tangent line lies above the curve. At point B, the graph is concave down and the tangent line lies above the curve. Point C is an inflection point, and the tangent line lies above the curve on the left and below the curve on the right. At point D, the graph is concave up and the tangent line lies below the curve. At point E, the tangent line cannot be drawn because the curve is not continuous there.

Refer to the graph.
(a) Is the graph concave up, concave down, or neither at points A, B, and C?
(b) If a tangent line cannot be drawn at a particular point, explain why not. Should the tangent lines lie above or below the curve at each of the remaining points?

2—3.2—ANSWER: At point A, the tangent line cannot be drawn because secant lines drawn with points on the right of A and on the left of A do not approach the same slope. At point B, the tangent line cannot be drawn because secant lines drawn with points on the right of B and on the left of B do not approach the same slope. At point C, the graph is concave up and the tangent line lies below the curve.

31. At which of the labeled points on the graph
(a) Is the curve concave up?
(b) Does the tangent line lie above the curve?
(c) Can the tangent line not be drawn? Explain.

2—3.2—ANSWER: (a) The curve is concave up at point E. (b) The tangent line lies above the curve at point C. (c) The tangent line cannot be drawn at point B because secant lines drawn with points on the left of B and on the right of B do not approach the same slope.
2. Refer to the graph.
   (a) At each labeled point on the graph, determine whether the instantaneous rate of change is positive, negative, or zero.
   (b) Is the graph steeper at point $A$ or point $C$?
   (c) Is the graph steeper at point $D$ or point $E$?
   (d) At which labeled point is the curve the steepest?

2—3.2—ANSWER: (a) The instantaneous rate of change is negative at points $A$, $D$, and $E$, positive at point $C$, and zero at point $B$. (b) The graph is steeper at point $A$ than at point $C$. (c) The graph is steeper at point $E$ than at point $D$. (d) The graph is steepest at point $E$.

3. Refer to the graph.
   (a) At each labeled point on the graph, determine whether the instantaneous rate of change is positive, negative, or zero.
   (b) Is the graph steeper at point $C$ or point $E$?
   (c) Is the graph steeper at point $B$ or point $C$?
   (d) Do any of the labeled points appear to be inflection points? If so, which points?
   (e) At which labeled points does the line tangent to the curve lie above the curve?

2—3.2—ANSWER: (a) The instantaneous rate of change is zero at point $A$, negative at points $B$ and $E$, and positive at points $C$ and $D$. (b) The graph is steeper at point $E$ than at point $C$. (c) The graph is steeper at point $C$ than at point $B$. (d) Points $B$, $C$, and $E$ appear to be inflection points. (e) The tangent line lies above the curve at points $A$, $B$ and $D$.

4. Discuss the slope of the graph.

1—3.2—ANSWER: The slope is always negative. From left to right, the magnitude of the slope starts out large and decreases as $x$ increases; the slope starts out very small and gradually becomes closer and closer to zero.

5. Discuss the slope of the graph.

1—3.2—ANSWER: The slope is always positive. The slope has the greatest magnitude at the left of the graph and the slopes decrease as $x$ increases. The slope is least at the right of the graph.
6. Discuss the slope of the graph.

1—3.2—ANSWER: From left to right, the slope starts out positive and large, decreases to zero at the top of the graph, and then becomes negative.

7. Discuss the slope of the graph.

1—3.2—ANSWER: The slope is always negative. From left to right, the slope starts out small, decreases to where it is the least, and then slowly increases to become closer and closer to zero; the magnitude of the slope starts out small, increases to where it is the largest, and then decreases to become small again.

8. Discuss the slope of the graph.

1—3.2—ANSWER: The slope of the graph is always negative. From left to right, the slope starts out small and gradually increases, becoming closer and closer to zero; the magnitude of the slope starts out large and becomes smaller and smaller, approaching zero.

9. Use a carefully drawn tangent line to estimate the slope at point P.

1—3.2—ANSWER: Slope at P ≈ −6 (Answers may vary.)
2—3.2—ANSWER: Slope at $Q \approx -2$ (Answers may vary.)

(a) Use a carefully drawn tangent line to estimate the slope at point $C$.
(b) Use a carefully drawn tangent line to estimate the slope at point $D$.
(c) If the vertical axis measures the profit, in thousands of dollars, from ticket sales and the horizontal axis measures the price of a ticket in dollars, what are the units of measure of the answer to part b?

2—3.2—ANSWER: (a) Slope at $C \approx -43$, Answers may vary. (b) Slope at $D \approx 65$, Answers may vary. (c) The units are thousand dollars per dollar.

The table gives the population of Dayton, OH, for selected years from 1950 through 1994. (Source: U.S. Bureau of the Census)

<table>
<thead>
<tr>
<th>Year</th>
<th>Population</th>
</tr>
</thead>
<tbody>
<tr>
<td>1950</td>
<td>243,872</td>
</tr>
<tr>
<td>1970</td>
<td>243,023</td>
</tr>
<tr>
<td>1980</td>
<td>193,536</td>
</tr>
<tr>
<td>1990</td>
<td>182,005</td>
</tr>
<tr>
<td>1994</td>
<td>178,540</td>
</tr>
</tbody>
</table>

(a) Use the data to calculate the change in population during the following intervals: (i) 1950 through 1970, (ii) 1970 through 1990, and (iii) 1990 through 1994.
(b) Express the changes in part a as percentage changes.
(c) Use the data to calculate the average rates of change for the same time intervals.
(d) What would be needed in order to calculate the instantaneous rate of change of population in 1970?

2—3.2—ANSWER: (a) (i) -849 people, (ii) -61,018 people, (iii) -3465 people; (b) (i) About -0.3%, (ii) About -25.1%, (iii) About -1.9%; (c) (i) -42.45 people per year; (ii) -3050.9 people per year, (iii) -866.25 people per year; (d) We would need a continuous graph modeling the data so that we could sketch a tangent line at 1970 and find its slope.

Southwest Airline Company’s revenue between 1990 and 1998 can be described by the function $R(t) = 0.165t - 0.226$ billion dollars when $t$ thousand people are employed by this company. (Source: Hoover’s Online Capsules) What was the instantaneous rate of change of Southwest Airlines revenue when 19,300 people were employed by the company?

2—3.2—ANSWER: 0.165 billion dollars per thousand people employed
The graph shows the number of Apple Computer, Inc. employees from 1988 through 1998. The slope at point A (year is 1990) is about 1. The slope at point B (year is 1994) is approximately -1.01. (Source: Based on data from Hoover’s Online Capsules)
(a) What should be the units on the slope at A and B?
(b) What is the instantaneous rate of change of the number of employees at point B?
(c) How rapidly was the number of employees changing in 1990?

2—3.2—ANSWER: (a) Thousands of employees per year; (b) Approximately -1.01 thousand employees per year; (c) Increasing by about 1000 employees per year

The graph shows the United States budget receipts from individual income taxes from 1990 through 2000. The slope at point A is approximately 21.54. The slope at point B is about 86.85. (Source: Based on data from the U.S. Department of the Treasury)
(a) What should be the units on the slope at A and B?
(b) What is the instantaneous rate of change of receipts at point A?
(c) How rapidly were receipts changing in 1998?
(d) Were receipts from individual income taxes increasing more rapidly at point A or B?

2—3.2—ANSWER: (a) Billions of dollars per year; (b) About $21.54 billion per year; (c) Receipts were increasing by about $86.85 billion per year in 1998; (d) Point B

The daily revenue of a campus courier service as a function of the predicted daily high temperature is shown in the graph. The slopes (in descending order) at the labeled points are 63.5, -30.2, and -123.9. (At A, temperature = 10°F; at B, temperature = 50°F; at C, temperature = 90°F.)
(a) What should be the units on the slope?
(b) Match the slopes with the points A, B, and C.
(c) How quickly is revenue changing with respect to temperature at 50°F?
(d) What is the instantaneous rate of change of revenue at 90°F?
(e) What is the slope of the tangent line at 10°F?

2—3.2—ANSWER: (a) Dollars per degree Fahrenheit; (b) A: -123.9 dollars per °F; B: -30.2 dollars per °F; C: 63.5 dollars per °F; (c) Decreasing by $30.2 per °F; (d) $63.5 per °F; (e) -123.9 dollars per °F
**Derivatives**

47. Suppose that \( T(x) \) equals the boiling point of water in degrees Celsius at altitude \( x \) feet. What are the units on \( \frac{dT}{dx} \)?

1—3.3—ANSWER: Degrees Celsius per foot

48. Let \( R(t) \) be the revenue in millions of dollars of a large corporation \( t \) years after 1990. What are the units on \( \frac{dR}{dt} \)?

1—3.3—ANSWER: Millions of dollars per year

49. Let \( B(h) \) be the cumulative number of books printed and bound on a production line \( h \) hours after the printing job is started. What are the units on \( B'(h) \)?

1—3.3—ANSWER: Books per hour

50. Let \( E(t) \) be the number of people employed by a certain company after \( t \) years.
   (a) Is it possible for \( E(t) \) to be negative? Explain.
   (b) What are the units on \( E'(t) \)?
   (c) Is it possible for \( E'(t) \) to be negative? Explain.

1—3.3—ANSWER: (a) No, it is not possible to have a negative number of employees; (b) Employees per year; (c) Yes, the number of employees could decrease, which would give a negative rate of change.

51. Let \( B(x) \) the systolic blood pressure (in mm Hg) of a patient \( t \) hours after surgery.
   (a) Is it possible for \( B(x) \) to be negative? Explain.
   (b) What are the units on \( B'(x) \)?
   (c) Is it possible for \( B'(x) \) to be negative? Explain.

1—3.3—ANSWER: (a) No, it is not possible to have a negative blood pressure; (b) mm Hg per hour; (c) Yes, if the patient's blood pressure falls, the rate of change in blood pressure would be negative.

52. Let \( P(x) \) be the price per share (in dollars) of Intel stock after \( x \) days.
   (a) Is it possible for \( P(x) \) to be negative? Explain.
   (b) What are the units on \( \frac{dP}{dx} \)?
   (c) Is it possible for \( \frac{dP}{dx} \) to be negative? Explain.

1—3.3—ANSWER: (a) No, it is not possible to have a negative stock price; (b) Dollars per share per day; (c) Yes, if the price of the stock decreases, then the rate of change of price per share would be negative.
52. Suppose that $P(h)$ is the profit (in hundreds of dollars) made by a certain electronics store by selling $h$ refrigerators. Interpret $P'(10) = 0.25$.

1—3.3—ANSWER: When the store sells 10 refrigerators, profit is increasing by $25 per refrigerator.

53. Suppose that $S(t)$ is the number of stores operated by a large discount department store chain $t$ years after 1998. Interpret the following:

(a) $S(3) = 2816$
(b) $S'(3) = 26$
(c) $\frac{dS}{dt} = -7$ when $s = 1$

2—3.3—ANSWER: (a) In 2001, the department store chain operated 2816 stores; (b) In 2001, the number of stores operated by the chain was increasing by 26 stores per year; (c) In 1999, the number of stores operated by the chain was decreasing by 7 stores per year.

54. Suppose that $C(f)$ is the number of children that register for summer day camp when the registration fee is set at $f$ dollars for a week of camp. Interpret the following:

(a) $C(100) = 500$
(b) $\frac{dC}{df} = -2$ when $f = 100$
(c) $C'(400) = -60$

2—3.3—ANSWER: (a) When the fee is set at $100 per week, 500 children register for camp; (b) When the fee is $100 per week, the number of children registered is decreasing by 2 children per dollar of registration fee; (c) When the fee is $400 per week, the number of children registered is decreasing by 60 children per dollar of registration fee.

55. On the basis of the information that follows, sketch a possible graph of $f$ with input $x$ when $-4 \leq x \leq 8$.

- $f(-1) = 0, f(0) = 5, f(5) = 0$
- $f''(2) = 0$
- The graph of $f$ has no concavity changes.
- $f(x) < 0$ when $x > 5$

3—3.3—ANSWER: See graph. (Answers may vary.)
56. On the basis of the following information, sketch a possible graph of \( g \) with input \( x \).
- \( g(4) = 3 \)
- \( \frac{d}{dx}[g(x)] = -0.75 \)

2—3.3—ANSWER: See graph.

7. On the basis of the following information, sketch a possible graph of \( c \) with input \( t \) for \( 0 \leq t \leq 6 \).
- \( c(3) = 4, c(5) = -4 \)
- \( \frac{dc}{dt} = 0 \) when \( t = 3 \); otherwise, \( c'(t) \) is negative for \( 0 \leq t \leq 6 \)

2—3.3—ANSWER: See graph.

8. Based on the following information, sketch a possible graph of a function \( p \) with input \( x \).
- \( p(1) = 1 \)
- \( p'(y) = 1 \) for \( y > 1 \)
- \( p'(y) \) is negative for \( y < 1 \)
- \( \lim_{y \to 1^-} p(y) = 1 \)
- \( \lim_{y \to 1^+} p(y) = 1 \)
- \( \frac{dp}{dy} \) does not exist when \( y = 1 \)

3—3.3—ANSWER: See graph.

9. For each of the following, discuss the graph of a function satisfying the stated condition.
(a) The function \( N \) with input \( r \) is such that \( N'(r) = 0 \) for all values of \( r \).
(b) The function \( Q \) with input \( s \) is such that \( Q'(s) > 0 \) for all values of \( s \).

2—3.3—ANSWER: (a) The graph of \( N \) is a horizontal line. (b) From left to right, the graph of \( Q \) is always increasing.
60. Based on the following information, sketch a possible graph of a function $w$ with input $x$.
- $w(4) = 3$
- $\frac{dw}{dx} = 1$ for $x > 1$
- $\lim_{x \to 1^-} w(x) = 0.22$
- $\lim_{x \to 1^+} w(x) = 1$

3—3.3—ANSWER: See graph.

61. Suppose that $D(t)$ gives the number of kilometers from the station that a train has traveled after $t$ hours. Interpret the following:
(a) $D(1) = 90$
(b) $D'(2) = 40$
(c) $\frac{dD}{dt} = 0$ when $t = 2.25$
(d) $D'(2.75) = 60$
(e) $D(3.5) = 240$

2—3.3—ANSWER: (a) After 1 hour, the train has traveled 90 km; (b) When the train has been traveling for 2 hours, its speed is 40 km per hour; (c) Two hours and 15 minutes into the trip, the train has stopped (because its speed in km per hour is 0); (d) At 2 hours and 45 minutes, the train is moving again at a speed of 60 km per hour; (e) After 3.5 hours, the train has traveled 240 km.

62. A soft drink bottling plant has a 60,000 gallon holding tank for fructose syrup. Suppose that $V(m)$ give the volume of fructose syrup in gallons held by the tank $m$ minutes after the first shift begins working at the plant. Interpret the following:
(a) $V(0) = 55,000$
(b) $V'(45) = -80$
(c) $V'(120) = 0$
(d) $V'(210) = 20$
(e) $V(240) = 52,340$

2—3.3—ANSWER: (a) At the beginning of the shift, the tank holds 55,000 gallons of fructose syrup; (b) After 45 minutes, the tank is emptying at a rate of 80 gallons per minute; (c) After 120 minutes, the volume held by the tank is neither increasing nor decreasing; (d) After 210 minutes, the volume of fructose syrup in the tank is increasing by 20 gallons per minute; (e) After 240 minutes, the tank holds 52,340 gallons of syrup.
Let $T(x)$ be the temperature in degrees Fahrenheit in Cincinnati, OH, $x$ hours after midnight.

(a) Is it possible for $T(x)$ to be negative? Explain.
(b) Is it possible for $T'(x)$ to be negative? Explain.
(c) What do we know about the temperature at noon if $T'(12) = -2$?
(d) What do we know about the temperature if $\frac{dT}{dx} = 0$ at 3 p.m.?

2—3.3—ANSWER: (a) Yes, it is possible for the temperature to be below 0°F; (b) Yes, it is possible for the derivative to be negative if the temperature drops as the day goes on; (c) If $T'(12) = -2$, the temperature is dropping at a rate of 2°F per hour at noon; (d) The temperature is neither increasing nor decreasing at 3 p.m. if the derivative is 0 at that point.

Let $f(x)$ be the number of packages misdirected by a shipping company when the company handles $x$ million packages per day.

(a) What are the units on $\frac{df}{dx}$?
(b) Is it possible for $f(x)$ to be negative? Explain.
(c) Is it possible for $f'(x)$ to be negative? Does it make practical sense for $f'(x)$ to be negative? Explain.

3—3.3—ANSWER: (a) (Misdirected) packages per million packages (handled) per day; (b) No, it is not possible for the number of misdirected packages to be less than 0; (c) Yes, a negative derivative would mean that the number of misdirected packages is declining; however, the opposite scenario—that the number of misdirected packages increases as the number of packages handled increases—would make more sense.

The graph shows the time in minutes needed to thoroughly bake a pizza when the oven is set on $x$ degrees Fahrenheit.

(a) Estimate the average rate of change between 250°F and 400°F. Interpret your answer.
(b) Estimate the instantaneous rate of change at 225°F. Interpret your answer.
(c) Estimate the instantaneous rate of change at 350°F. Interpret your answer.

2—3.3—ANSWER: (Answers may vary on all parts.) (a) The average rate of change between 250°F and 400°F can be approximated by the slope of a secant line; the required baking time declines by approximately 0.33 minute (or 20 seconds) per degree of baking temperature between 250°F and 400°F; (b) The instantaneous rate of change at 225°F can be approximated by the slope of a tangent line at 225°F; the required baking time declines by approximately 0.8 minute per degree of baking time at 225°F; (c) The instantaneous rate of change at 350°F can be approximated by the slope of a tangent line at 350°F; the required baking time declines by approximately 0.25 minute per degree of baking time at 350°F.
A function \( P \), with input \( x \) years after 1980, that describes the number of bomb threats received against U.S. aircraft between 1980 and 1996 is graphed in the figure. (Source: Based on data in the Statistical Abstract, 1998)

(a) Estimate and interpret the derivative \( P'(10) \).
(b) Estimate and interpret \( P(14) \).
(c) Approximately when is \( P'(x) = 0 \)?
(d) Estimate and interpret the derivative \( \frac{dP}{dx} \)

at \( x = 2 \).

2—3.3—ANSWER: (Answers may vary on all parts.) (a) \( P'(10) \approx 19.6 \); In 1990 the number of bomb threats received against U.S. aircraft was increasing by about 19.6 threats per year; (b) \( P(14) \approx 278 \); About 278 bomb threats against U.S. aircraft were received in 1994. (c) \( P'(x) = 0 \) in about 1987. (d) \( \frac{dP}{dx} \approx -32.7 \); Bomb threats received against U.S. aircraft were decreasing by about 32.7 threats per year in 1982.

The table shows the earnings per share \( E(t) \) of Wendy's International, Inc., stock in year \( t \) from 1990 through 1999. (Source: Hoover's Online Guide)

<table>
<thead>
<tr>
<th>Year</th>
<th>1990</th>
<th>1991</th>
<th>1993</th>
<th>1995</th>
<th>1997</th>
<th>1999</th>
</tr>
</thead>
<tbody>
<tr>
<td>Earnings (cents per share)</td>
<td>41</td>
<td>52</td>
<td>67</td>
<td>88</td>
<td>97</td>
<td>132</td>
</tr>
</tbody>
</table>

(a) Use a symmetric difference quotient to estimate the rate of change in earnings per share in 1993. Interpret your answer.
(b) Use a symmetric difference quotient to estimate the derivative \( E'(1997) \). Interpret your answer.

2—3.3—ANSWER: (a) Rate of change \( \approx 9 \) cents per share per year; In 1993, earnings were increasing by about 9 cents per share per year; (b) Rate of change \( \approx 11 \) cents per share per year; In 1997, earnings were increasing by 11 cents per share per year.

The table shows the number of packages \( P(x) \) delivered by a shipping company each week \( x \) after a strike by delivery drivers.

<table>
<thead>
<tr>
<th>Week</th>
<th>0</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
</tr>
</thead>
<tbody>
<tr>
<td>Packages (thousands)</td>
<td>180</td>
<td>130</td>
<td>95</td>
<td>70</td>
<td>60</td>
<td>62</td>
<td>75</td>
</tr>
</tbody>
</table>

(a) Use a symmetric difference quotient to estimate the rate of change in packages delivered in the first week. Interpret your answer.
(b) Use a symmetric difference quotient to estimate the derivative \( P'(5) \). Interpret your answer.

2—3.3—ANSWER: (a) One week after the strike, the number of packages delivered was decreasing by 42.5 thousand packages per week; (b) After 5 weeks, the number of packages delivered was increasing by 7.5 thousand packages per week.
The table shows the number of bacteria $B(x)$ in a biological experiment after $x$ hours.

<table>
<thead>
<tr>
<th>Hours</th>
<th>0</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
</tr>
</thead>
<tbody>
<tr>
<td>Bacteria (hundreds)</td>
<td>0.23</td>
<td>0.54</td>
<td>13.5</td>
<td>32.0</td>
<td>76.5</td>
<td>184.3</td>
<td>443.6</td>
</tr>
</tbody>
</table>

(a) Use a symmetric difference quotient to estimate the rate of change in bacteria after 2 hours. Interpret your answer.

(b) Use a symmetric difference quotient to estimate the derivative $B'(4)$. Interpret your answer.

(c) What do your answers in parts a and b tell you about the growth of the bacteria?

2—3.3—ANSWER: (a) After 2 hours, the number of bacteria is growing at a rate of 15.73 hundred bacteria per hour; (b) After 4 hours, the number of bacteria is growing at a rate of 76.15 hundred bacteria per hour; (c) Not only is the bacteria increasing in number, the rate of growth of the bacteria is increasing as well.

In 1999, Reebok International Ltd. posted $165 million in net income, and net income was decreasing by about $58 million per year. (Source: Hoover’s Company Profiles)

(a) What is the rate of change of net income for 1995?

(b) What is the percentage rate of change of net income for 1995?

1—3.3—ANSWER: (a) −$58 million per year; (b) About −35.2% per year

In 1995, McDonald’s Corporation made $9795 million in sales, and sales were increasing by about $1183 million per year. (Source: Hoover’s Company Profiles)

(a) What is the rate of change of sales for 1995?

(b) What is the percentage rate of change of sales for 1995?

(c) Use the rate of change of sales to approximate McDonald’s Corporation sales in 1996.

2—3.3—ANSWER: (a) $1183 million per year; (b) About 12.1% per year; (c) Approximately $10,978

The value of a college student’s savings at the beginning of her sophomore year is $1340, and her savings were increasing by about $30 per month.

(a) What is the rate of change of the student’s savings with respect to time?

(b) What is the percentage rate of change of the student’s savings?

1—3.3—ANSWER: (a) $30 per month; (b) About 2.24% per month

In 1995, the number of U.S. households subscribing to cable was 58.3 million and was increasing by 2.6 million per year. (Source: based on data from Televison and Cable Factbook)

(a) What is the rate of change of cable subscribers for 1995?

(b) What is the percentage rate of change of cable subscribers for 1995?

(c) Use the derivative in 1995 to approximate the number of subscribing households in 1996.

2—3.3—ANSWER: (a) 2.6 million subscribers per year; (b) About 4.46% per year; (c) About 60.9 million households
List as many facts as you can about the slope of the graph.

1—3.4—ANSWER: The slope is always negative and is constant.

List as many facts as you can about the slope of the graph.

1—3.4—ANSWER: The slope is always positive. The magnitude of the slope is close to zero at the left, and the magnitude becomes larger as $x$ increases.

List as many facts as you can about the slope of the graph.

1—3.4—ANSWER: The slope is always negative. The magnitude of the slope is large at the left and the magnitude is close to zero at the right.

List as many facts as you can about the slope of the graph. Then, based on those facts, sketch the slope graph of the function.

2—3.4—ANSWER: The slope is always positive and is constant. See graph.
16. List as many facts as you can about the slope of the graph. Then, based on those facts, sketch the slope graph of the function.

2—3.4—ANSWER: The slope is negative to the left of $A$ and positive to the right of $A$. The slope is zero at $A$. See graph.

77. List as many facts as you can about the slope of the graph. Then, based on those facts, sketch the slope graph of the function.

2—3.4—ANSWER: The slope is positive to the left of $A$ and negative to the right of $A$. The slope is zero at $A$. See graph.

78. List as many facts as you can about the slope of the graph. Then, based on those facts, sketch the slope graph of the function.

2—3.4—ANSWER: The slope is always positive. The magnitude of the slope is very large at the left and becomes closer to 0 as $x$ increases. See graph.
The scatter plot shows single parent families as a percent of all families for the years 1960 through 1998. (Based on data in Index of Leading Cultural Indicators)
(a) Sketch a smooth curve through the scatter plot with no more inflection points than suggested by the data. What type of model would be appropriate for these data?
(b) Estimate the slope of your smooth curve at any inflection points.
(c) Sketch a rate-of-change graph. Label the units on both axes of your graph.

3—3.4—ANSWER: (a) See graph. A cubic function seems more appropriate than a logistic function because the last data point indicates a downward trend. (b) An inflection point occurs at about 18.9 years after 1960. The slope at that point is approximately 0.83 percentage points per year. (Answers may vary.) (c) See graph.

The graph shows the average size of a U.S. farm from 1940 through 1998. (Source: Based on data from the National Agricultural Statistics Service)
(a) Estimate the instantaneous rates of change in 1940, 1960, 1980, and 1990.
(b) Based on your answers to part a, sketch a rate-of-change graph. Label the units on the axes.
(c) When was the average farm size growing most rapidly? What is this point called?

3—3.4—ANSWER: (a) About 3.2 acres per year in 1940, about 7.6 acres per year in 1960, about 5.1 acres per year in 1980, and about 1.3 acres per year in 1990; (Answers may vary.) (b) See graph; (c) At an input value of about 62.7 or the year 1963. This is an inflection point.
The graph shows the number of McDonald's Corp. employees (in thousands) between 1987 and 1998. (Source: Based on data in Hoover’s Online Guide)

(a) Sketch a smooth curve through the scatter plot with no more inflection points than suggested by the data.
(b) Where does the smooth curve have slope zero?
(c) Estimate the slope of your smooth curve at any inflection points.
(d) Sketch a rate-of-change graph. Label the units on both axes of your graph.

3—3.4—ANSWER: (a) See graph; (b) At inputs of about 1.7 and about 4.6; (c) Inflection points occur at inputs of approximately 3, where the slope is about −6.2 thousand employees per year, and approximately at 9, where the slope is about 29.8 thousand employees per year. (Answers may vary.) (d) See graph.

The graph shows the net profit for Papa John’s International, Inc., a pizza restaurant chain, from 1990 through 1999. (Source: Based on data from Hoover’s Online Guide)

(b) Based on your answers to part (a), sketch a rate-of-change graph. Label the units on the axes.

3—3.4—ANSWER: (a) About $0.05 million per year in 1991, about $2.9 million per year in 1993, about $5.8 million per year in 1995, about $8.6 million per year in 1997, and about $11.5 million per year in 1999; (Answers may vary on all slopes.) (b) See graph.
(a) Use the given set of axes to carefully sketch a graph of the function \( f(x) = 0.8x^2 + 2x + 3 \) for \(-6 \leq x \leq 6\).
(b) Estimate the slope of the graph at \( x = -6, -3, 0, 3, 6 \).
(c) Where does the minimum output occur?
(d) Based on your answers to parts \( a \) and \( b \), sketch the rate-of-change graph. Label each axis appropriately.

2–3.4—ANSWER: (a) See graph. (b) About \(-7.6\) at \( x = -6 \), about \(-2.8\) at \( x = -3 \), about \( 2 \) at \( x = 0 \), about \( 6.8 \) at \( x = 3 \), and about \( 11.6 \) at \( x = 6 \) (Answers for all slopes may vary.) (c) At about \( x = -1.25 \); (d) See graph.

(a) ![Graph of f(x)](attachment:image1)
(b) ![Graph of f'(x)](attachment:image2)

8.4. (a) Use the given set of axes to sketch a graph of the function \( f(x) = -1.4x^2 + 5x + 7 \) for \(-1 \leq x \leq 5\).
(b) Estimate the slope of the graph at \( x = 0, 1, 2, 3, \) and \( 4 \).
(c) Where does the maximum output occur?
(d) Based on your answers to parts \( a \) and \( b \), sketch a rate-of-change graph. Label each axis appropriately.

2–3.4—ANSWER: (a) See graph. (b) About \( 5 \) at \( x = 0 \), about \( 2.2 \) at \( x = 1 \), about \(-0.6 \) at \( x = 2 \), about \(-3.4 \) at \( x = 3 \), and about \(-6.2 \) at \( x = 4 \) (Answers for all slopes may vary.) (c) At about \( x = 1.75 \); (d) See graph.

(a) ![Graph of f(x)](attachment:image3)
(b) ![Graph of f'(x)](attachment:image4)
Determining Change: Derivatives

Numerically Finding Slopes

The average local monthly bill for cellular telephone subscribers between 1990 and 1998 can be modeled by the function $B(x) = 128.759x^{-0.269}$ dollars when there are $x$ million subscribers. (Source: Based on data from the Cellular Telecommunications Industry Association)

(a) Numerically estimate the limit of slopes of secant lines on the graph of $B$ between the point corresponding to $x = 85$ and close points to the left of $x = 85$.

(b) Numerically estimate the limit of slopes of secant lines on the graph of $B$ between the point corresponding to $x = 85$ and close points to the right of $x = 85$.

(c) How rapidly is the local monthly bill for cellular telephone subscribers changing when there are 85 million subscribers?

1—4.1—ANSWER: (a), (b) See table. (c) Decreasing by $0.12$ dollar per million subscribers

<table>
<thead>
<tr>
<th>Input of close point on left</th>
<th>Secant line slope</th>
<th>Input of close point on right</th>
<th>Secant line slope</th>
</tr>
</thead>
<tbody>
<tr>
<td>84.9</td>
<td>$-0.1234$</td>
<td>85.1</td>
<td>$-0.1232$</td>
</tr>
<tr>
<td>84.99</td>
<td>$-0.1233$</td>
<td>85.01</td>
<td>$-0.1233$</td>
</tr>
<tr>
<td>84.999</td>
<td>$-0.1233$</td>
<td>85.001</td>
<td>$-0.1233$</td>
</tr>
<tr>
<td>(a) Trend =</td>
<td>$-0.12$</td>
<td>(b) Trend =</td>
<td>$-0.12$</td>
</tr>
</tbody>
</table>

The number of discharges per 1000 people in non-Federal short-stay hospitals (based on the civilian population as of July 1 of the indicated year) between 1980 and 1996 is given by the function $A(x) = 162.1485 - 19.9363 \ln x$ discharges per 1000 people where $x$ is the number of years after 1979. (Source: Based on data in the National Hospital Discharge Survey)

(a) Numerically estimate the limit of slopes of secant lines on the graph of $A$ between the point corresponding to $x = 11$ and close points to the left of $x = 11$.

(b) Numerically estimate the limit of slopes of secant lines on the graph of $A$ between the point corresponding to $x = 11$ and close points to the right of $x = 11$.

(c) How rapidly was the number of discharges per 1000 people changing in 1990?

1—4.1—ANSWER: (a), (b) See table. (c) Decreasing by 1.8 discharges per 1000 people per year

<table>
<thead>
<tr>
<th>Input of close point on left</th>
<th>Secant line slope</th>
<th>Input of close point on right</th>
<th>Secant line slope</th>
</tr>
</thead>
<tbody>
<tr>
<td>10.9</td>
<td>$-1.82$</td>
<td>11.1</td>
<td>$-1.80$</td>
</tr>
<tr>
<td>10.99</td>
<td>$-1.81$</td>
<td>11.01</td>
<td>$-1.81$</td>
</tr>
<tr>
<td>10.999</td>
<td>$-1.81$</td>
<td>11.001</td>
<td>$-1.81$</td>
</tr>
<tr>
<td>(a) Trend =</td>
<td>$-1.8$</td>
<td>(b) Trend =</td>
<td>$-1.8$</td>
</tr>
</tbody>
</table>
The graph shows the body-mass index (BMI) for women weighing 120 pounds and ranging in height from 50 through 75 inches. The equation is \[ \text{BMI} = \frac{83.7001674}{h^2} \], where \( h \) is a woman's height in inches. (Source: Based on data from Good Housekeeping, August 1997)

(a) Carefully sketch the tangent line where \( h = 60 \), and estimate its slope.

(b) Use the equation to estimate numerically the slope of the line tangent to the graph at \( h = 60 \).

(c) Interpret your answer to part b as a rate of change.

2—4.1—ANSWER: (a) Approximately \(-0.78\) points per inch (Answers will vary.) (b) See table. The slope of the tangent line is about \(-0.775\) points per inch. (c) At a height of 60 inches, the BMI index is decreasing by approximately 0.775 points per inch.

<table>
<thead>
<tr>
<th>Input of close point on left</th>
<th>Secant line slope</th>
<th>Input of close point on right</th>
<th>Secant line slope</th>
</tr>
</thead>
<tbody>
<tr>
<td>59.9</td>
<td>-0.7769</td>
<td>60.1</td>
<td>-0.7730</td>
</tr>
<tr>
<td>59.99</td>
<td>-0.7751</td>
<td>60.01</td>
<td>-0.7748</td>
</tr>
<tr>
<td>59.999</td>
<td>-0.7750</td>
<td>60.001</td>
<td>-0.7749</td>
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<td>59.9999</td>
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<td>60.0001</td>
<td>-0.7749</td>
</tr>
<tr>
<td>Trend = -0.775</td>
<td></td>
<td>Trend = -0.775</td>
<td></td>
</tr>
</tbody>
</table>

The graph shows the average size of a U.S. farm from 1940 through 1999. The equation is \[ S(x) = -0.00335x^3 + 0.6234x^2 - 30.8270x + 621.4540 \] acres \( x \) years since 1900. (Source: Based on data from the National Agricultural Statistics Service)

(a) Carefully sketch the tangent line where \( x = 70 \), and estimate its slope.

(b) Use the equation to estimate numerically the slope of the line tangent to the graph at \( x = 70 \).

(c) Interpret your answer to part b as a rate of change.

2—4.1—ANSWER: (a) Approximately 7.20 acres per year (Answers will vary.) (b) See table. The slope of the tangent line is about 7.20 acres per year. (c) In 1970, the average size of a U.S. farm was increasing by approximately 7.20 acres per year.

<table>
<thead>
<tr>
<th>Input of close point on left</th>
<th>Secant line slope</th>
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<tr>
<td>69.9</td>
<td>7.211</td>
<td>70.1</td>
<td>7.195</td>
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<td>69.99</td>
<td>7.204</td>
<td>70.01</td>
<td>7.203</td>
</tr>
<tr>
<td>69.999</td>
<td>7.204</td>
<td>70.001</td>
<td>7.203</td>
</tr>
<tr>
<td>69.9999</td>
<td>7.204</td>
<td>70.0001</td>
<td>7.203</td>
</tr>
<tr>
<td>Trend = 7.20</td>
<td></td>
<td>Trend = 7.20</td>
<td></td>
</tr>
</tbody>
</table>
The graph shows how the number of Campbell Soup employees (in thousands) changed between 1990 and 1998. The equation is

\[ E(t) = -0.2297t^3 + 2.3326t^2 - 7.1067t + 49.9310 \]

thousand employees \( t \) years since 1990. (Source: Based on data from Hoover’s Online Guide)

(a) Using only the graph, estimate how rapidly the number of employees was changing in 1994.

(b) Use the equation to estimate numerically, to the nearest hundred employees, the rate of change of the number of employees in 1994.

(c) Which of the two methods (part a or part b) is more accurate? Explain.

2—4.1—ANSWER: (a) Approximately 0.53 thousand employees per year or 530 employees per year (Answers will vary.) (b) See table; The number of employees was increasing by about 0.53 thousand employees per year in 1994. (c) Part b is more accurate because of the error introduced when drawing of the tangent line and estimating the values in part a.

<table>
<thead>
<tr>
<th>Input of close point on left</th>
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<th>Input of close point on right</th>
<th>Secant line slope</th>
</tr>
</thead>
<tbody>
<tr>
<td>3.9</td>
<td>0.568</td>
<td>4.1</td>
<td>0.483</td>
</tr>
<tr>
<td>3.99</td>
<td>0.532</td>
<td>4.01</td>
<td>0.524</td>
</tr>
<tr>
<td>3.999</td>
<td>0.528</td>
<td>4.001</td>
<td>0.528</td>
</tr>
<tr>
<td>3.9999</td>
<td>0.528</td>
<td>4.0001</td>
<td>0.528</td>
</tr>
<tr>
<td>3.99999</td>
<td>0.528</td>
<td>4.00001</td>
<td>0.528</td>
</tr>
<tr>
<td>Trend =</td>
<td>0.53</td>
<td>Trend =</td>
<td>0.53</td>
</tr>
</tbody>
</table>

The percentage of U.S. car sales that were midsize cars from 1980 through 1998, modeled by \( P(x) = 0.0819x^2 - 1.0338x + 42.958 \) percent \( x \) years since 1980, is graphed in the figure. (Source: Based on data from the American Automobile Manufacturers Association)

(a) Use the graph to estimate \( \frac{dP}{dx} \) when \( x = 10 \).

(b) Use the equation to numerically estimate \( P'(10) \).

(c) Interpret your answer to part b.

2—4.1—ANSWER: (a) Approximately 0.6 percentage point per year (Answers will vary.) (b) See table. (c) The percentage of U.S. car sales that were midsize cars was increasing by about 0.6 percentage point per year in 1990.

<table>
<thead>
<tr>
<th>Input of close point on left</th>
<th>Secant line slope</th>
<th>Input of close point on right</th>
<th>Secant line slope</th>
</tr>
</thead>
<tbody>
<tr>
<td>9.99</td>
<td>0.603</td>
<td>10.01</td>
<td>0.605</td>
</tr>
<tr>
<td>9.999</td>
<td>0.604</td>
<td>10.001</td>
<td>0.604</td>
</tr>
<tr>
<td>9.9999</td>
<td>0.604</td>
<td>10.0001</td>
<td>0.604</td>
</tr>
<tr>
<td>Trend =</td>
<td>0.6</td>
<td>Trend =</td>
<td>0.6</td>
</tr>
</tbody>
</table>
The graph shows the amount of outstanding consumer credit, in billions of dollars, in the U.S. between 1990 and 1999. The equation is \( C(t) = -2.0304t^3 + 32.1015t^2 - 56.2477t + 794.1771 \) billion dollars \( t \) years after 1990. (Source: Based on data from *Statistical Abstract, 1998*)

(a) Use the graph to estimate \( C'(8) \).

(b) Use the equation to estimate numerically, to the nearest tenth of a billion dollars, \( C'(8) \).

(c) Interpret your answer to part b.

(d) Discuss the advantages and disadvantages of using the two methods (in parts a and b) for finding derivatives.

3—4.1—ANSWER: (a) Approximately 67.5 billion dollars per year (Answers will vary.) (b) See table; \( C'(8) \approx 67.5 \) billion dollars per year in 1998. (c) The amount of outstanding consumer credit in the U.S. was increasing by about 67.5 billion dollars per year in 1998. (d) The method in part a is slightly faster than that in part b but it can be inaccurate. The part b method can be a little tedious but is more accurate.

<table>
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<tr>
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<th>Secant line slope</th>
<th>Input of close point on right</th>
<th>Secant line slope</th>
</tr>
</thead>
<tbody>
<tr>
<td>7.9</td>
<td>67.18</td>
<td>8.1</td>
<td>65.85</td>
</tr>
<tr>
<td>7.99</td>
<td>67.70</td>
<td>8.01</td>
<td>67.37</td>
</tr>
<tr>
<td>7.999</td>
<td>67.55</td>
<td>8.001</td>
<td>67.52</td>
</tr>
<tr>
<td>7.9999</td>
<td>67.54</td>
<td>8.0001</td>
<td>67.53</td>
</tr>
<tr>
<td>7.999999</td>
<td>67.53</td>
<td>8.000001</td>
<td>67.53</td>
</tr>
<tr>
<td>Trend =</td>
<td>67.5</td>
<td>Trend =</td>
<td>67.5</td>
</tr>
</tbody>
</table>

The yearly amount of time spent on-line by an average U.S. resident 18 years of age or older between 1990 and 2001 can be modeled by \( I(x) = \frac{38.6141}{1 + 294.2397e^{-0.8497x}} + 0.5395 \) hours where \( x \) is the number of years after 1990. (Source: Based on data in *Statistical Abstract, 1998*)

(a) According to the model, how much time was spent on-line by the average U.S. resident 18 years of age or older in 2000?

(b) Use the equation to numerically estimate the derivative of \( I(x) \) when \( x = 10 \).

(c) Interpret your answer to part a.

(d) Is \( I'(8) \) more than, less than, or about the same as \( I'(10) \)? Give a reason for your answer based on the graph of \( I \) rather than on a numerical answer.

2—4.1—ANSWER: (a) About 37 hours; (b) The yearly amount of time an average person spent on-line was increasing by \( I'(10) \approx 1.75 \) hours per year. (c) The yearly amount of time spent on-line by an average U.S. resident 18 years of age or older was increasing by about 1.75 hours per year in 2000. (d) The line tangent to the graph of \( I \) is steeper at \( x = 8 \) than at \( x = 10 \) and the graph of \( I \) is increasing. Therefore, \( I'(8) \) is greater than \( I'(10) \).
The amount of energy imported into the United States from 1960 through 1994 can be modeled by \( E(x) = 4.623(1.047855^x) \) quadrillion Btu \( x \) years since 1960. (Source: Based on data from the U.S. Department of Energy)

(a) Use the equation to numerically estimate the derivative of \( E \) when \( x = 16 \).

(b) Interpret your answer to part \( a \).

1—4.1—ANSWER: (a) \( E'(16) \approx 0.4565 \) quadrillion Btu per year; (b) In 1976 the amount of energy imported into the United States was increasing by approximately 0.4565 quadrillion Btu per year.

The number of cases of a certain infectious disease reported in the United States can be modeled by \( D(t) = 225(0.743^t) \) thousand cases after \( t \) years.

(a) Use the equation to numerically estimate the derivative of \( D \) when \( t = 2 \).

(b) Interpret your answer to part \( a \).

1—4.1—ANSWER: (a) \( D'(2) \approx -36.898 \) thousand cases per year; (b) After 2 years, the number of cases of the disease reported in the United States was decreasing by approximately 36.898 thousand cases per year.

The number of bank failures in the United States from 1988 through 1995 can be modeled by

\[
F(x) = \frac{234.779}{1 + 0.06e^{0.88918x}} \] failures \( x \) years after 1988. (Source: Based on data from the Federal Deposit Insurance Corporation)

(a) Use the equation to numerically estimate the derivative of \( F \) when \( x = 4 \).

(b) Interpret your answer to part \( a \).

1—4.1—ANSWER: (a) \( F'(4) \approx -45.33 \) failures per year; (b) In 1992 the number of bank failures in the United States was decreasing by approximately 45.33 failures per year.

The table shows the number of seat belt warnings issued by the Ohio State Highway Patrol in selected years from 1991 through 1996. (Source: Ohio State Highway Patrol)

<table>
<thead>
<tr>
<th></th>
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<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>Seat belt warnings</td>
<td>45,837</td>
<td>25,222</td>
<td>18,267</td>
<td>12,375</td>
</tr>
</tbody>
</table>

(a) Use a symmetric difference quotient to approximate the rate of change of the number of seat belt warnings issued by the Ohio State Highway Patrol in 1995.

(b) Fit a linear model to these data.

(c) What is the slope of the equation in part \( b \)? Compare this slope to your answer in part \( a \), and discuss the information each contributes to your understanding of the change in the number of seat belt warnings issued.

2—4.1—ANSWER: (a) \( -6423.5 \) warnings per year; (b) \( W(x) = -6749.571x + 45,673.964 \) seat belt warnings \( x \) years since 1991; (c) Slope = \(-6749.571\) seat belt warnings per year; This slope estimate has a greater magnitude than that given by the symmetric difference quotient. The part \( c \) slope is the rate of change that would have occurred if the data had been perfectly linear. The answer to part \( a \) is a better picture of the rate of change in 1995.
The table shows the cumulative number of paper grocery bags used at a grocery store over a period of several weeks.

<table>
<thead>
<tr>
<th>Week</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
<th>7</th>
<th>8</th>
</tr>
</thead>
<tbody>
<tr>
<td>Cumulative number of paper bags used</td>
<td>1253</td>
<td>2360</td>
<td>3462</td>
<td>4560</td>
<td>5665</td>
<td>6765</td>
<td>7868</td>
<td>8970</td>
</tr>
</tbody>
</table>

(a) Use a symmetric difference quotient to approximate the rate of change of paper bag usage after 6 weeks.
(b) Find a linear model to fit the data in the table.
(c) What is the slope of the equation from part b? Compare this slope to your answer in part a, and discuss the information each contributes to your understanding of the change in the number of paper bags used.

3—4.1—ANSWER: (a) 1101.5 paper bags per week; (b) \( b(x) = 1102.060x + 153.607 \) paper bags after \( x \) weeks; (c) Slope = 1102.060 paper bags per week, which is quite close to the answer found in part a. This slope is the rate of change that would have occurred if the data had been perfectly linear. The answer to part a is a (slightly) better picture of the rate of change after 6 weeks.

Based on data from 1988 through 1998, the prices for 3-ounce servings of boneless choice beef are as given in the table. (Source: USDA: Cattle and Beef Industry Statistics, March 1999)

<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>Price (dollars per serving)</td>
<td>0.50</td>
<td>0.56</td>
<td>0.57</td>
<td>0.56</td>
<td>0.55</td>
<td>0.58</td>
</tr>
</tbody>
</table>

(a) Use a symmetric difference quotient to approximate the rate of change of price in 1990.
(b) Find a cubic model for these data.
(c) Based on a graph of the function in part b, estimate when the price of a 3-ounce serving was decreasing most rapidly.
(d) Use the equation to estimate numerically how rapidly the price was changing in 1990.
(e) Compare your answers to parts a and d. Discuss the advantages and disadvantages of using each of the two methods.

2—4.1—ANSWER: (a) Approximately $0.02 per serving per year; (b) A cubic function is given by the equation \( P(s) = (5.9028 \cdot 10^{-4})s^3 - 0.0099s^2 + 0.0478s + 0.4998 \) dollars per serving \( s \) years after 1988; (c) The inflection point appears to occur in about 1994. (d) See table. Tangent line slope \( \approx 2 \) cents per serving per year; (e) In this case, both methods give the same answer to the nearest cent. While the symmetric difference quotient gives a quick estimate, the numerical estimate may in general be more accurate provided that the model fits the data well (as is the case here). However, the numerical estimate takes longer.

<table>
<thead>
<tr>
<th>Input of close point on left</th>
<th>Secant line slope</th>
<th>Input of close point on right</th>
<th>Secant line slope</th>
</tr>
</thead>
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<tr>
<td>1.9</td>
<td>0.0159</td>
<td>2.1</td>
<td>0.0146</td>
</tr>
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<td>1.99</td>
<td>0.0153</td>
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<td>0.0152</td>
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<td>1.999</td>
<td>0.0153</td>
<td>2.001</td>
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<td>1.9999</td>
<td>0.0153</td>
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<tr>
<td>Trend =</td>
<td>0.02</td>
<td>Trend =</td>
<td>0.02</td>
</tr>
</tbody>
</table>
Algebraically Finding Slopes

At a certain university, the number of tickets sold for a home basketball game can be modeled by \( t(w) = 0.0038w^2 + 1.414w + 4 \) hundred tickets where \( w \) is the winning percentage of the home team, \( 0 \leq w \leq 100 \).

(a) Find the number of tickets sold when the team has won \( w = 60 \) percent of its games.
(b) Write an expression for the number of tickets sold when \( w = 60 + h \).
(c) Write a simplified formula for the slope of the secant line connecting the points at \( w = 60 \) and \( w = 60 + h \).
(d) What is the limiting value of the slope formula in part c as \( h \) approaches 0?
(e) Interpret your answer to part d.

2—4.2—ANSWER: (a) \( t(60) = 102.52 \) hundred tickets;
(b) \( t(60 + h) = 0.0038(60 + h)^2 + 1.414(60 + h) + 4 = 0.0038h^2 + 1.87h + 102.52; \)
(c) \( \frac{t(60+h) - t(60)}{(60+h) - 60} = \frac{0.0038h^2 + 1.87h + 102.52 - 102.52}{h} = \frac{h(0.0038h + 1.87)}{h}; \)
(d) \( \lim_{h \to 0} \frac{h(0.0038h + 1.87)}{h} = \lim_{h \to 0} (0.0038h + 1.87) = 1.87 \) hundred tickets per percentage point;
(e) When the home team has won 60% of its games, the number of tickets sold for a home game is increasing by 187 tickets per percentage point increase in the home team’s winning percentage.

The consumer price index (CPI) for medical care from 1970 through 1990 is given by the function \( f(x) = 0.227x^2 + 1.945x + 33.185 \) points \( x \) years since 1970. (Source: Based on data from the Bureau of Labor Statistics)
(a) Find the CPI for medical care in 1988 when \( x = 18 \).
(b) Write an expression for the CPI for medical care when \( x = 18 + h \).
(c) Write a simplified formula for the slope of the secant line connecting the points at \( x = 18 \) and \( x = 18 + h \).
(d) What is the limiting value of the slope formula in part c as \( h \) approaches 0?
(e) Interpret your answer to part d.

2—4.2—ANSWER: (a) \( f(18) = 141.743 \) points;
(b) \( f(18 + h) = 0.227(18 + h)^2 + 1.945(18 + h) + 33.185 = 0.227h^2 + 10.117h + 141.743; \)
(c) \( \frac{f(18+h) - f(18)}{(18+h) - 18} = \frac{0.227h^2 + 10.117h + 141.743 - 141.743}{h} = \frac{h(0.227h + 10.117)}{h}; \)
(d) \( \lim_{h \to 0} \frac{h(0.227h + 10.117)}{h} = \lim_{h \to 0} (0.227h + 10.117) = 10.117 \) points per year;
(e) In 1988 the CPI for medical care was increasing by 10.117 points per year.

Use the Four-Step Method to show that the derivative of \( y = 4x - 1 \) is \( \frac{dy}{dx} = 4. \)

1—4.2—ANSWER: i. \( f(x) = 4x - 1; \) ii. \( f(x + h) = 4(x + h) - 1 = 4x + 4h - 1; \)
iii. \( \frac{f(x + h) - f(x)}{x + h - x} = \frac{4x + 4h - 1}{h} = \frac{4h}{h}; \) iv. \( \lim_{h \to 0} \frac{4h}{h} = \lim_{h \to 0} 4 = 4. \) Therefore, \( \frac{dy}{dx} = 4. \)
The daily revenue of a student-owned campus courier service can be modeled by the function \( r(t) = 1.171t^2 - 147.286t + 4770 \) dollars, where \( t \) is the predicted daily high temperature in degrees Fahrenheit.

(a) Find the daily revenue when the temperature is \( t = 80 \).
(b) Write an expression for the daily revenue for \( t = 80 + h \).
(c) Write a simplified formula for the slope of the secant line connecting the points at \( t = 80 \) and \( t = 80 + h \).
(d) What is the limiting value of the slope formula in part (c) as \( h \) approaches 0?
(e) Interpret your answer to part (d).

2.4.2—ANSWER: (a) \( r(80) = 481.52 \) dollars;
(b) \( r(80 + h) = 1.171(80 + h)^2 - 147.286(80 + h) + 4770 = 1.17h^2 + 40.074h + 481.52 \);
(c) \( \frac{r(80+h) - r(80)}{h} = \frac{1.17h^2 + 40.074h + 481.52 - 481.52}{h} = \frac{h(1.17h + 40.074)}{h} \);
(d) \( \lim_{h \to 0} \frac{h(1.17h + 40.074)}{h} = \lim_{h \to 0} (1.17h + 40.074) = 40.074 \) dollars per degree Fahrenheit;
(e) When the predicted daily high temperature is 80°F, daily revenue is increasing by $40.07 per degree Fahrenheit increase in the predicted daily high temperature.

The Summer Olympics are held every 4 years. The winning time in the women’s 200-meter run event in the 20th (held in 1972) through the 24th Olympiad (held in 1988) can be modeled by the function \( w(x) = -0.0543x^2 + 2.1206x + 1.7203 \) seconds in the \( x \)th Olympiad. (Source: Based on data from The World Almanac and Book of Facts, 1997)

(a) Find the winning time in the 23rd Olympiad (in 1984).
(b) Write an expression for the winning time in the \( x \)th Olympiad, where \( x = 23 + h \).
(c) Write a simplified formula for the slope of the secant line connecting the points at \( x = 23 \) and \( x = 23 + h \).
(d) What is the limiting value of the slope formula in part (c) as \( h \) approaches 0?
(e) Interpret your answer to part (d).

2.4.2—ANSWER: (a) \( w(23) = 21.7694 \) seconds;
(b) \( w(23 + h) = -0.0543(23 + h)^2 + 2.1206(23 + h) + 1.7203 = -0.0543h^2 - 0.3772h + 21.7694 \);
(c) \( \frac{w(23 + h) - w(23)}{23 + h - 23} = \frac{-0.0543h^2 - 0.3772h + 21.7694 - 21.7694}{h} = h(-0.0543h - 0.3772) \);
(d) \( \lim_{h \to 0} \frac{h(-0.0543h - 0.3772)}{h} = \lim_{h \to 0} (-0.0543h - 0.3772) = -0.3772 \) seconds per Olympiad;
(e) At the 23rd Olympiad, the winning time in the women’s 200-meter run was decreasing by 0.3772 seconds per Olympiad.

Use the Four-Step Method to show that the derivative of \( y = 5.3 - 7x \) is \( \frac{dy}{dx} = -7 \).

1.4.2—ANSWER: i. \( f(x) = 5.3 - 7x \); ii. \( f(x + h) = 5.3 - 7(x + h) = 5.3 - 7x - 7h \);
iii. \( \frac{f(x + h) - f(x)}{x + h - x} = \frac{5.3 - 7x - 7h - (5.3 - 7x)}{h} = \frac{-7h}{h} = -7 \); iv. \( \lim_{h \to 0} \frac{-7h}{h} = \lim_{h \to 0} -7 = -7 \), so \( \frac{dy}{dx} = -7 \).
The yearly consumer spending per person for television subscription video services between 1994 and 2001 can be described by \( G(x) = -0.6687x^2 + 22.3761x + 31.2664 \) dollars, where \( x \) is the number of years after 1990. (Source: Based on data in *Statistical Abstract*, 1998)

(a) Find the budget receipts when \( x = 7 \).

(b) Write an expression for the budget receipts when \( x = 7 + h \).

(c) Write a simplified formula for the slope of the secant line connecting the points at \( x = 7 \) and \( x = 7 + h \).

(d) What is the limiting value of the slope formula in part (c) as \( h \) approaches 0?

(e) Interpret your answer to part (d).

\[ 2—4.2—\text{ANSWER:} \quad (a) \ G(7) = 155.1238; \]

\[ (b) \ G(7 + h) = -0.6687(7 + h)^2 + 22.3761(7 + h) + 31.2664 = -0.6687h^2 + 13.0143h + 155.1238 \]

\[ (c) \ \frac{G(7 + h) - G(7)}{(7 + h) - 7} = \frac{-0.6687h^2 + 13.0143h + 155.1238 - 155.1238}{h} = \frac{h(-0.6687h + 13.0143)}{h} \]

\[ (d) \ \lim_{h \to 0} \frac{h(-0.6687h + 13.0143)}{h} = \lim_{h \to 0} (-0.6687h + 13.0143) \approx 13.02 \text{ per year}; \]

(e) The yearly consumer spending per person for television subscription video services were increasing by about $13.02 per year in 1997.

The table shows the size of the U.S. public debt from 1986 through 1992. (Source: U.S. Bureau of the Public Debt)

<table>
<thead>
<tr>
<th></th>
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<tbody>
<tr>
<td>Public debt (trillions of dollars)</td>
<td>2.125</td>
<td>2.350</td>
<td>2.602</td>
<td>2.857</td>
<td>3.233</td>
<td>3.665</td>
<td>4.065</td>
</tr>
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</table>

(a) Use a symmetric difference quotient to approximate the rate of change of the size of the U.S. public debt in 1990.

(b) Find a quadratic model to fit the data. Use an input equal to the number of years after 1980. Round off the quadratic function to 3 decimal places.

(c) Use the rounded function to find the size of the public debt in 1990.

(d) Use the rounded function to write a formula in terms of \( h \) for the size of the public debt a little after 1990.

(e) Write a simplified formula for the slope of the secant line connecting the point at 1990 and the point a little after 1990.

(f) What is the limiting value for the slope formula as \( h \) approaches 0?

(g) Interpret your answer to part (f).

\[ 2—4.2—\text{ANSWER:} \quad (a) \ \text{Rate of change} \approx 0.404 \text{ trillion per year}; \quad (b) \quad \text{The quadratic function is} \quad D(x) = 0.024x^2 - 0.108x + 1.915 \text{ trillion dollars} \times x \text{ years since 1980.} \quad (c) \quad D(10) = 3.235 \text{ trillion}; \]

\[ (d) \ D(10 + h) = 0.024(10 + h)^2 - 0.108(10 + h) + 1.915 = 0.024h^2 + 0.372h + 3.235 \]

\[ (e) \ \frac{D(10 + h) - D(10)}{(10 + h) - 10} = \frac{0.024h^2 + 0.372h + 3.235 - 3.235}{h} = \frac{h(0.024h + 0.372)}{h} \]

\[ (f) \ \lim_{h \to 0} \frac{h(0.024h + 0.372)}{h} = \lim_{h \to 0} (0.024h + 0.372) \approx 0.372 \text{ trillion per year}; \]

(g) The size of the U.S. public debt was increasing by about $0.372 trillion per year in 1990.
07. Use the Four-Step Method to show that the derivative of \( f(x) = -35.2 - 1.21x \) is \( f'(x) = -1.21 \).

2—4.2—ANSWER: i. \( f(x) = -35.2 - 1.21x \);
   ii. \( f(x + h) = -35.2 - 1.21(x + h) = -35.2 - 1.21x - 1.21h; \)
   iii. \( \frac{f(x + h) - f(x)}{x + h - x} = \frac{-35.2 - 1.21x - 1.21h - (-35.2 - 1.21x)}{h} = \frac{-1.21h}{h} \);
   iv. \( \lim_{h \to 0} \frac{-1.21h}{h} = -1.21 \). Thus, \( f'(x) = -1.21 \).

08. Use the Four-Step Method to show that the derivative of \( f(x) = 6.3x^2 \) is \( f'(x) = 12.6x \).

2—4.2—ANSWER: i. \( f(x) = 6.3x^2 \);
   ii. \( f(x + h) = 6.3(x + h)^2 = 6.3x^2 + 12.6xh + h^2; \)
   iii. \( \frac{f(x + h) - f(x)}{x + h - x} = \frac{6.3x^2 + 12.6xh + h^2 - 6.3x^2}{h} = \frac{h(12.6x + h)}{h} \);
   iv. \( \lim_{h \to 0} \frac{h(12.6x + h)}{h} = \lim_{h \to 0} (12.6x + h) = 12.6x \). Thus, \( f'(x) = 12.6x \).

09. Use the Four-Step Method to show that the derivative of \( y = 3x^2 + 5.4x + 1 \) is \( \frac{dy}{dx} = 6x + 5.4 \).

2—4.2—ANSWER: i. \( f(x) = 3x^2 + 5.4x + 1 \);
   ii. \( f(x + h) = 3(x + h)^2 + 5.4(x + h) + 1 = 3x^2 + 6xh + h^2 + 5.4x + 5.4h + 1; \)
   iii. \( \frac{f(x + h) - f(x)}{x + h - x} = \frac{3x^2 + 6xh + h^2 + 5.4x + 5.4h + 1 - (3x^2 + 5.4x + 1)}{h} = \frac{h(6x + h + 5.4)}{h}; \)
   iv. \( \lim_{h \to 0} \frac{h(6x + h + 5.4)}{h} = \lim_{h \to 0} (6x + h + 5.4) = 6x + 5.4 \). Thus, \( f'(x) = \frac{dy}{dx} = 6x + 5.4 \).

10. Use the Four-Step Method to find a derivative formula for \( y = \frac{-1}{x} \) is \( \frac{dy}{dx} = \frac{1}{x^2} \). [Hint: Multiply numerator and denominator of the slope formula by \( x(x + h) \).]

3—4.2—ANSWER: i. \( f(x) = \frac{-1}{x} \); ii. \( f(x + h) = \frac{-1}{x + h} \);
   iii. \( \frac{f(x + h) - f(x)}{x + h - x} = \frac{-1}{x + h} - \frac{-1}{x} \frac{x(x + h)}{h} = \frac{-x(x + h)}{hx(x + h)} = \frac{h}{hx(x + h)} \);
   iv. \( \lim_{h \to 0} \frac{h}{hx(x + h)} = \lim_{h \to 0} \frac{1}{x(x + h)} = \frac{1}{x^2} \). Thus, \( f'(x) = \frac{dy}{dx} = \frac{1}{x^2} \).
Simple Rate-of-Change Formulas

111. Find the derivative formula for each function.

(a) \( y = 5x - 16.1 \)  
(b) \( G(t) = 45.782 \)  
(c) \( P(x) = x^3 \)

1—4.3—ANSWER: (a) \( y' = 5 \); (b) \( G'(t) = 0 \); (c) \( P'(x) = 3x^2 \)

112. Find the derivative formula for each function.

(a) \( y = 32.354 - 1.673x \)  
(b) \( g(x) = -38.222 \)  
(c) \( T(m) = m^5 \)

1—4.3—ANSWER: (a) \( y' = -1.673 \); (b) \( g'(x) = 0 \); (c) \( T'(m) = 5m^4 \)

113. Find the rate-of-change formula for each function.

(a) \( y = 5x - 16.1 \)  
(b) \( B(x) = 9x^6 \)  
(c) \( R(w) = w^{-2} \)

1—4.3—ANSWER: (a) \( y' = 5 \); (b) \( B'(x) = 54x^5 \); (c) \( R'(w) = -2w^{-3} \)

114. Find the rate-of-change formula for each function.

(a) \( P(x) = \sqrt{x} - 5.09x \)  
(b) \( L(t) = 0.22t^{-6.3} \)  
(c) \( y = 1 + x^2 - \frac{3}{x} \)

1—4.3—ANSWER: (a) \( P'(x) = \frac{1}{2}x^{-1/2} - 5.09 \); (b) \( L'(t) = -1.386t^{-7.3} \); (c) \( y' = 2x + 3x^{-2} \)

115. (a) Sketch the slope graph for the function whose graph is shown in the figure.

(b) The equation of the function is \( f(x) = -2x^3 + 4 \). Give the slope equation.

1—4.3—ANSWER: (a) See graph. (b) \( f'(x) = -6x^2 \)
116. (a) Sketch the slope graph for the function whose graph is shown in the figure.
(b) The equation of the function is \( h(x) = \frac{3}{x} - 1.5 \). Give the slope equation.

1—4.3—ANSWER: (a) See graph. (b) \( h'(x) = -\frac{3}{x^2} \)

117. (a) Sketch the slope graph for the function whose graph is shown in the figure.
(b) The equation of the function is \( f(x) = -4.1x^2 + 2x + 1.8 \). Give the slope equation.

1—4.3—ANSWER: (a) See graph. (b) \( f'(x) = -8.2x + 2 \)

118. (a) Sketch the slope graph for the function whose graph is shown in the figure.
(b) The equation of the function is \( g(x) = 5 - \frac{2}{\sqrt{x}} \). Give the slope equation.

1—4.3—ANSWER: (a) See graph. (b) \( g'(x) = x^{-3/2} \)
120. The U.S. gross national product from 1960 through 1995 can be modeled by the function
\[ P(x) = 5.520x^2 - 661.028x + 20,285.401 \] billion dollars \( x \) years after 1900. (Source: Based on data from the U.S. Bureau of Economic Analysis)
(a) What was the gross national product in 1995?
(b) Find the rate of change of the gross national product in 1995.
(c) Find the percentage rate of change of the gross national product in 1995.

1—4.3—ANSWER: (a) \( P(95) = 7305.741 \) billion dollars; (b) \( P'(95) = 387.772 \) billion dollars per year; (c) \( \frac{P'(95)}{P(95)} \cdot 100\% \approx 5.3 \) percent per year

122. The Hertz Corporation revenue between 1987 and 1998 can be described by the function
\[ R(t) = 0.001396t^3 - 0.0502t^2 + 0.7679t - 1.5698 \] billion dollars where \( t \) is the number of years after 1980.
(a) Find a derivative formula for the given revenue function.
(b) How rapidly was Hertz's revenue changing in 1990? In 1997?

1—4.3—ANSWER: (a) \( R'(t) = 0.004188t^2 - 0.1004t + 0.7679 \) billion dollars per year \( t \) years after 1980; (b) \( R'(10) = 0.1827 \) billion dollars per year; \( R'(17) \approx 0.2714 \) billion dollars per year
Preprimary school enrollment of 3-year olds includes public and nonpublic nursery school and kindergarten programs. The preprimary school enrollment between (Source: Statistical Abstract, 1998)

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<tbody>
<tr>
<td>Enrollment of 3-year olds (thousands)</td>
<td>454</td>
<td>683</td>
<td>857</td>
<td>1035</td>
<td>1205</td>
<td>1489</td>
</tr>
</tbody>
</table>

(a) Find a function to fit the data.
(b) Estimate the 3-year enrollment preprimary enrollment in 1996.
(c) The 3-year enrollment in 1996 was 1,506,000 children. How close was your estimate in part b?
(d) Find a formula for the derivative of the function in part a.
(e) How quickly was the preprimary enrollment changing in 1990?

2—4.3—ANSWER: (a) $E(x) = 39.537x + 459.619$ thousand 3-year children in preprimary school $x$ years after 1970. (b) $E(26) = 1488$ thousand children; (c) The function in part a slightly underestimates the enrollment (by 18 thousand 3-year olds). (d) $E'(x) = 39.537$ thousand children per year $x$ years after 1970; (e) Increasing by 39,537 thousand children per year.

The table shows the poverty level in the United States for a family of four from 1991 through 1995. (Source: U.S. Department of Commerce)

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<tbody>
<tr>
<td>Poverty level (dollars)</td>
<td>13,924</td>
<td>14,335</td>
<td>14,763</td>
<td>15,141</td>
<td>15,569</td>
</tr>
</tbody>
</table>

(a) Find a model for the poverty level.
(b) Find a formula for the rate of increase of the poverty level over this period.
(c) How quickly was the poverty level increasing in 1995?

2—4.3—ANSWER: (a) $L(x) = 409.6x + 13,927.2$ dollars $x$ years after 1991; (b) $L'(x) = 409.6$ dollars per year $x$ years after 1991; (c) $L'(4) = 409.6$ dollars per year.

The table shows the total farm real estate debt in the United States from 1986 through 1992. (Source: U.S. Department of Agriculture)

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<tbody>
<tr>
<td>Debt (billions of dollars)</td>
<td>90.4</td>
<td>82.4</td>
<td>77.8</td>
<td>76.0</td>
<td>74.7</td>
<td>74.9</td>
<td>75.4</td>
</tr>
</tbody>
</table>

(a) Align the input as the number of years after 1986. Find a function $D$ to fit the data.
(b) Write the formula for the derivative of the model from part a.
(c) Use the result of part b to find $D'(1)$. Interpret your result.
(d) Use the result of part b to find $D'(6)$. Interpret your result.

2—4.3—ANSWER: (a) $D(x) = 0.804x^2 - 7.075x + 89.579$ billion dollars $x$ years after 1986; (b) $D'(x) = 1.607x - 7.075$ billion dollars per year $x$ years after 1986; (c) $D'(1) = -5.468$ billion dollars per year; The total U.S. farm real estate debt was decreasing by $5.468$ billion per year in 1987; (d) $D'(6) = 2.568$ billion dollars per year; The total U.S. farm real estate debt was increasing by about $2.568$ billion per year in 1992.
More Simple Rate-of-Change Formulas

Find the rate-of-change formula for each function.
(a) \( f(t) = t^3 \)
(b) \( J(x) = 5^x \)
(c) \( Q(w) = 4.7e^w - 2.3^w \)

1—4.4—ANSWER: (a) \( f'(t) = -3t^4 \); (b) \( J'(x) = (\ln 5)(5^x) \); (c) \( Q'(w) = 4.7e^w - (\ln 2.3)(2.3^w) \)

Find the slope formula for each function.
(a) \( f(x) = 2.419^x \)
(b) \( t(x) = 12.36 + 6.2 \ln x \)
(c) \( H(u) = 3e^u \)

1—4.4—ANSWER: (a) \( f'(x) = (\ln 2.419)(2.419^x) \); (b) \( t'(x) = 6.2 \left( \frac{1}{x} \right) \); (c) \( H'(u) = 3e^u \)

Find the derivative formula for each function.
(a) \( y = 5.3x^2 + \frac{87.41}{x^2} \)
(b) \( y = 6 \ln x - 2.3^x \)

1—4.4—ANSWER: (a) \( y' = 10.6x - \frac{174.82}{x^3} \); (b) \( y' = 6 \left( \frac{1}{x} \right) - (\ln 2.3)(2.3^x) \)

Find the derivative formula for each function.
(a) \( d(x) = 14.9e^x - \frac{27.3}{x} \)
(b) \( P(t) = 16.73 \ln t - 10.43t \)

1—4.4—ANSWER: (a) \( d'(x) = 14.9e^x + 27.3x^{-2} \); (b) \( P'(t) = 16.73 \left( \frac{1}{t} \right) - 10.43 \)

Find the rate-of-change formula for each function.
(a) \( g(x) = 81.2x^2 - 33.6 \ln x \)
(b) \( m(t) = 208e^t - 13\sqrt{t} \)

1—4.4—ANSWER: (a) \( g'(x) = 162.4x - 33.6 \left( \frac{1}{x} \right) \); (b) \( m'(t) = 208e^t - 6.5t^{-1/2} \)

Find the derivative formula for each function.
(a) \( Z(w) = 12(1.08^w) + 3w^3 - 14 \)
(b) \( P(x) = 16.2x^4 - 2.5x^2 + 7e^x - 9.2 \)

1—4.4—ANSWER: (a) \( Z'(w) = 12(\ln 1.08)(1.08^w) + 9w^2 \); (b) \( P'(x) = 64.8x^3 - 5x + 7e^x \)
Find the rate-of-change formula for each of the following functions.
(a) \( Q(t) = 10(3.054^t) - 28.903y \)
(b) \( A(t) = -3.671t^4 - 2.095 \ln t \)

1—4.4—ANSWER: (a) \( Q'(t) = 10(\ln 3.054)(3.054^t) - 28.903 \); (b) \( A'(t) = -14.684t^3 - 2.095 \left( \frac{1}{t} \right) \)

Find the derivative formula for each of the following functions.
(a) \( R(x) = 13.1x^3 + 2.9x - 6.8x - 17.4 - \frac{154}{x} \)
(b) \( I(t) = 500(1.0495^t) - 500e^t + e^2 \)

1—4.4—ANSWER: (a) \( R'(x) = 39.3x^2 + (\ln 2.9)(2.9x) - 6.8 + \frac{154}{x^2} \);
(b) \( I'(t) = 500(\ln 1.0495)(1.0495^t) - 500e^t \)

Find the derivative formula for each of the following functions.
(a) \( h(x) = 294.8 \ln x - 12.4 \)
(b) \( b(t) = 2.8(3.95^t) - 20e^t \)
(c) \( g(u) = 4u - 12.4 \left( \frac{1}{x} \right) + 23^4 \)

1—4.4—ANSWER: (a) \( h'(x) = 294.8 \left( \frac{1}{x} \right) \); (b) \( b'(t) = 2.8(\ln 3.95)(3.95^t) - 20e^t \);
(c) \( g'(u) = (\ln 4)(4^u) + 12.4x^{-2} \)

Find the derivative formula for \( A(t) = 2000 \left( 1 + \frac{0.065}{26} \right)^{26t} \). (Hint: Write the function in the form \( A(t) = ab^t \).)

2—4.4—ANSWER: \( A(t) \approx 2000(1.06707^t) \), so \( A'(t) \approx 2000(\ln 1.06707)(1.06707^t) \)

The value of a $3000 investment made at 4.8% interest compounded quarterly is given by the formula \( A(t) = 3000 \left( 1 + \frac{0.048}{4} \right)^{4t} \) dollars, where \( t \) is the number of years after the $3000 was initially invested.

(a) What is the value of the investment after 8 years?
(b) Rewrite the formula in the form \( A(t) = ab^t \).
(c) Find a derivative formula for \( A \).
(d) How rapidly is the value of the investment increasing after 8 years? After 15 years?

2—4.4—ANSWER: (a) \( A(8) \approx 4354.38 \); (b) The rewritten formula is \( A(t) \approx 3000(1.048871^t) \);
(c) \( A'(t) \approx 3000(\ln 1.048871)(1.048871^t) \) dollars per year \( t \) years after the money was initially invested; (d) \( A'(8) \approx 209.67 \) per year; \( A'(15) \approx 292.82 \) per year
The Chain Rule

The attendance at an amusement park increases by 700 people for each one degree Fahrenheit increase in daily high temperature. Daily revenue increases by $28 for each additional visitor to the park. How quickly is the park’s daily revenue increasing per degree Fahrenheit of daily high temperature?

1—4.5—ANSWER: (700 people per degree)($28 per person) = $19,600 per degree Fahrenheit of daily high temperature.

A certain student’s GPA increases by 0.001 points per 20 hours of study. The chance of getting a job upon graduation increases by 10 percentage points per 0.2 point increase in GPA. How quickly is this student’s chance of getting a job upon graduation increasing per hour of study?

1—4.5—ANSWER: The chance of this student getting a job upon graduation is increasing by \( \frac{0.001 \text{ GPA point}}{20 \text{ hours}} \cdot \frac{10 \text{ percentage points}}{0.2 \text{ GPA point}} = 0.0025 \text{ percentage points per hour of study}. \)

The function \( p \) with input \( t \) and the function \( t \) with input \( w \) can be composed to form the function with output \( p(t(w)) \). If \( t(-7) = 12 \), \( p(12) = -10 \), \( t'(-7) = 3 \), and \( p'(12) = -1 \), give the values of

(a) \( p(t(-7)) \)
(b) \( \frac{dt}{dw} \) when \( w = -7 \)
(c) \( \frac{dp}{dt} \) when \( t = 12 \)
(d) \( \frac{dp}{dw} \) when \( w = -7 \)

1—4.5—ANSWER: (a) \( p(t(-7)) = p(12) = -10 \); (b) \( \frac{dt}{dw} = 3 \); (c) \( \frac{dp}{dt} = -1 \); (d) \( \frac{dp}{dw} = \frac{dp}{dt} \cdot \frac{dt}{dw} = \)

(3)(-1) = -3

Let \( y \) be a function of \( t \) and let \( m \) be a function whose input corresponds to the output of \( y \). If \( m(2) = 21 \), \( m'(2) = -5 \), \( y(15) = 2 \), and \( y'(15) = 6 \), give the values of

(a) \( m(y(15)) \)
(b) \( \frac{dm}{dy} \) when \( y = 2 \)
(c) \( \frac{dy}{dt} \) when \( t = 15 \)
(d) \( \frac{dm}{dt} \) when \( t = 15 \)

1—4.5—ANSWER: (a) \( m(y(15)) = m(2) = 21 \); (b) \( \frac{dm}{dy} = -5 \); (c) \( \frac{dy}{dt} = 6 \); (d) \( \frac{dm}{dt} = \frac{dm}{dy} \cdot \frac{dy}{dt} = \)

(-5)(6) = -30
11. Let \( P \) dollars be the price of an airline ticket on a certain route when the demand for the ticket is \( d \) tickets. Let \( D \) tickets be the demand for the airline ticket on the \( t \)th day of the current year. On the 328th day of the year (in late November), the demand for the airline ticket was 800 tickets, and that number was increasing by 12 tickets per day. The rate of change of the price of the ticket is a constant $3 per ticket. Identify the following quantities on the 328th day of the year, and write a sentence interpreting each value.

(a) \( D(328) \)
(b) \( \frac{dD}{dt} \)
(c) \( \frac{dP}{dd} \)
(d) \( \frac{dP}{dt} \)

2—4.5—ANSWER: (a) \( D(328) = 800 \) tickets; On the 328th day of the year, the demand on a certain route was 800 tickets. (b) \( \frac{dD}{dt} = 12 \) tickets per day; On the 328th day of the year, the demand for tickets on a certain route was increasing by 12 tickets per day; (c) \( \frac{dP}{dd} = 3 \) per ticket; At any level of demand, the price of the ticket is increasing by $3 per ticket demanded; (d) \( \frac{dP}{dt} = (3 \text{ dollars per ticket})(12 \text{ tickets per day}) = 36 \) per day; On the 328th day of the year, the price of an airline ticket on a certain route was increasing by 36 per day.

A college student takes a summer job in a local factory as a toy assembler. The student does piecework, i.e., is paid per completed piece. Let \( P(x) \) be the number of pieces the student has assembled \( x \) hours from the start of the student’s shift. Let \( T(p) \) be the student’s earnings for the day when the student has completed \( p \) pieces. One hour after the shift starts, the student has assembled 25 pieces and the rate of change at that time is 29 pieces per hour. After assembling 25 pieces, the student has earned $10 and the student’s earnings are increasing by $0.42 per completed piece. Identify the following values at one hour after the start of the shift, and write a sentence interpreting each value.

(a) \( P(1) \) 
(b) \( T(25) \) 
(c) \( \frac{dT}{dx} \) 
(d) \( \frac{dT}{dp} \) 

2—4.5—ANSWER: (a) \( P(1) = 25 \) pieces; After the first hour of the shift, the student has assembled 25 pieces. (b) \( T(25) = 10 \); When the student has assembled 25 pieces, the student has earned $10; (c) \( \frac{dT}{dx} = 29 \) pieces per hour; The number of pieces assembled is increasing by 29 pieces per hour one hour after the start of the shift. (d) \( \frac{dT}{dp} = 0.42 \) per completed piece; The student’s earnings are increasing at a rate of $0.42 per completed piece one hour after the shift started. (e) \( \frac{dT}{dx} = (29 \text{ pieces per hour})(0.42 \text{ per completed piece}) = 12.18 \) per hour; One hour after the start of the shift, the student’s earnings are increasing by $12.18 per hour.
The distance traveled by the sound of an evacuation siren \( s \) seconds after it was activated is given by \( d(s) = 1129.7s \) feet. The circular area over which the siren can be heard is given by \( A(d) = \pi d^2 \) square feet. Find the value of each of the following when \( s = 5 \) seconds.

(a) \( d(s) \)
(b) \( A(d) \)
(c) \( d'(s) \)
(d) \( A'(d) \)
(e) \( \frac{dA}{ds} \)

Interpret this result.

3—4.5—ANSWER: (a) \( d(5) = 5648.5 \) feet; (b) When \( s = 5 \), \( d = 5648.5 \), so \( A(5648.5) = 100,234,248.6 \) square feet; (c) \( d'(s) = 1129.7 \) feet per second, so \( d'(5) = 1129.7 \) feet per second; (d) \( A'(d) = 2\pi d \) square feet of hearing area per foot of diameter, so \( A'(5648.5) \approx 35,490.6 \) square feet of hearing area per foot of diameter; (e) \( \frac{dA}{ds} = (2\pi d)(1129.7) = 2\pi(1129.7s)(1129.7) = 2,552,444.18\pi s \) square feet per second. When \( s = 5 \) seconds, \( \frac{dA}{ds} = 2,552,444.18\pi(5) \approx 40,093,699 \) square feet per second. Five seconds after the siren was activated, the area over which the siren can be heard is increasing by approximately 40,093,699 square feet per second.

Rewrite \( r(t) = 6t^2 - 3 \) and \( f(y) = 2y + 9 \) as a single composite function. Then find the derivative of the composite function.

1—4.5—ANSWER: \( r(t) = (2y + 9)^2 - 3 \), \( \frac{dr}{dy} = 2(6)(2y + 9)(2) = 24(2y + 9) \)

Rewrite \( w(x) = \frac{x}{14} \) and \( x(a) = 3a^2 - 7a + 2 \) as a single composite function. Then find the derivative of the composite function.

1—4.5—ANSWER: \( w(x(a)) = \frac{3a^2 - 7a + 2}{14} \), \( \frac{dw}{da} = \frac{1}{14} (6a - 7) \)

Rewrite \( q(m) = \sqrt{m} \) and \( m(x) = 4x^2 - 33.7x + 2 \) as a single composite function. Then find the derivative of the composite function.

1—4.5—ANSWER: \( q(m(x)) = \sqrt{4x^2 - 33.7x + 2} \), \( \frac{dq}{dx} = 0.5(4x^2 - 33.7x + 2)^{-1/2} (8x - 33.7) \)

Rewrite \( w(r) = -6e^r \) and \( r(x) = -2x \) as a single composite function. Then find the derivative of the composite function.

1—4.5—ANSWER: \( w(r(x)) = -6e^{-2x} \), \( \frac{dw}{dx} = -6(-2)e^{-2x} = 12e^{-2x} \)
147. Rewrite \( t(y) = 12e^y \) and \( y(s) = 5s^2 + 2s \) as a single composite function. Then find the derivative of the composite function.

\[
1-4.5-\text{ANSWER: } t(y(s)) = 12e^{5s^2 + 2s}; \quad \frac{dt}{ds} = 12(10s + 2)e^{5s^2 + 2s}
\]

148. Rewrite \( a(m) = -7e^m + 2m \) and \( m(x) = 5 - 3x \) as a single composite function. Then find the derivative of the composite function.

\[
1-4.5-\text{ANSWER: } a(m(x)) = -7e^5 - 3x + 2(5 - 3x); \quad \frac{da}{dx} = -7(-3)e^5 - 3x - 6 = 21e^5 - 3x - 6
\]

149. Consider the functions \( m(y) = 6y^2 - 3 \) and \( y(t) = 2t + 9 \). Find the value of each of the following when \( t = -2 \).

(a) \( m(y(t)) \)
(b) \( \frac{dm}{dy} \)
(c) \( \frac{dy}{dt} \)
(d) \( \frac{dm}{dt} \)

\[
1-4.5-\text{ANSWER: } (a) m(y(-2)) = m(5) = 147; \quad (b) \frac{dm}{dy} = 12y = 60; \quad (c) \frac{dy}{dt} = 2; \quad (d) \frac{dm}{dt} = \frac{dm}{dy} \cdot \frac{dy}{dt} = 120
\]

150. For the composite function \( f(x) = (13.2x^3 - 7.1x)^2 \), identify an inside and an outside function. Then find the derivative of the composite function.

\[
1-4.5-\text{ANSWER: } \text{Inside function: } u = 13.2x^3 - 7.1x; \text{outside function: } f = u^2; \quad f'(x) = 2(13.2x^3 - 7.1x)(39.6x^2 - 7.1)
\]

151. For the composite function \( f(x) = (5x^4 - 16)^3 \), identify an inside and an outside function. Then find the derivative of the composite function.

\[
1-4.5-\text{ANSWER: } \text{Inside function: } u = 5x^4 - 16; \text{outside function: } f = u^3; \quad f'(x) = 3(5x^4 - 16)^2(20x^3)
\]

152. For the composite function \( f(x) = (-9x^2 + 5x)^{-1} \), identify an inside and an outside function. Then find the derivative of the composite function.

\[
1-4.5-\text{ANSWER: } \text{Inside function: } u = -9x^2 + 5x; \text{outside function: } f = u^{-1}; \quad f'(x) = -1(-9x^2 + 5x)^{-2}(-18x + 5)
\]
For the composite function \( g(x) = \frac{1}{5.2x^2 - 3x} \), identify an inside and an outside function. Then find the derivative of the composite function.

1—4.5—ANSWER: Inside function: \( u = 5.2x^2 - 3x \); outside function: \( g = u^{-1} \);
\[ g'(x) = -1(5.2x^2 - 3x)^{-2}(10.4x - 3) \]

For the composite function \( g(x) = \sqrt{11 - 2x^2 + x^3} \), identify an inside and an outside function. Then find the derivative of the composite function.

1—4.5—ANSWER: Inside function: \( u = 11 - 2x^2 + x^3 \); outside function: \( g = u^{1/2} \);
\[ g'(x) = \frac{1}{2}(11 - 2x^2 + x^3)^{-1/2}(-4x + 3x^2) \]

For the composite function \( f(x) = 3 - \ln(2x + 5) \), identify an inside and an outside function. Then find the derivative of the composite function.

1—4.5—ANSWER: Inside function: \( u = 2x + 5 \); outside function: \( f = 3 - \ln u \);
\[ f'(x) = (-1)\left(\frac{1}{2x + 5}\right)(2) = \frac{-2}{2x + 5} \]

For the composite function \( f(x) = 2 \ln(15x^2) \), identify an inside and an outside function. Then find the derivative of the composite function.

1—4.5—ANSWER: Inside function: \( u = 15x^2 \); outside function: \( f = 2 \ln u \);
\[ f'(x) = 2\left(\frac{1}{15x^2}\right)(30x) = \frac{4}{x} \]

For the composite function \( f(x) = (\ln x)^3 \), identify an inside and an outside function. Then find the derivative of the composite function.

1—4.5—ANSWER: Inside function: \( u = \ln x \); outside function: \( f = u^3 \); \( f'(x) = \frac{3(\ln x)^2}{x} \)

For the composite function \( g(x) = \frac{-45}{6 - 15.9x} \), identify an inside and an outside function. Then find the derivative of the composite function.

1—4.5—ANSWER: Inside function: \( u = 6 - 15.9x \); outside function: \( g = -45u^{-1} \);
\[ g'(x) = -45(-1)(6 - 15.9x)^{-2}(-15.9) \]
For the composite function \( h(x) = \frac{74.862}{1 + 7.89e^{0.022x}} \), identify an inside and an outside function.

Then find the derivative of the composite function.

2—4.5—ANSWER: Inside function: \( u = 1 + 7.89e^{0.022x} \) (inside: \( w = 0.022x \); outside: \( u = 1 + 7.89e^{w} \)); outside function: \( h = 74.862u^{-1} \); \( h'(x) = 74.862(-1)(u^{-2})(7.89e^{w})(0.022) = 74.862(-1)(1 + 7.89e^{0.022x})^{-2}(7.89e^{0.022x})(0.022) \)

160. Write the derivative formula for each of the following functions.

(a) \( y = 98 - \ln(5x + 2) \)
(b) \( p(w) = (3w^5 - 6w^2 + 9)^2 \)
(c) \( R(t) = e^{2t} + \frac{3}{t^2} + 3.5^{0.8} \)

1—4.5—ANSWER: (a) \( y' = \frac{-5}{5x+2} \); (b) \( p'(w) = 2(3w^5 - 6w^2 + 9)(15w^4 - 12w) \);
(c) \( R'(t) = 2e^{2t} - 6t^{-3} + 0 \)

161. Write the derivative formula for each of the following functions.

(a) \( R(t) = \ln(6t^2) \)
(b) \( A(u) = 2.57 - (3u^5 - 6)^4 \)
(c) \( p(w) = 12e^{2w} + 3 \)
(d) \( g(x) = \frac{2}{1 + 5e^{-0.2x}} \)

1—4.5—ANSWER: (a) \( R'(t) = \frac{2}{t} \); (b) \( A'(u) = -4(3u^5 - 6)^3(15u^4) \); (c) \( p'(w) = 24e^{2w} + 3 \);
(d) \( g'(x) = 2e^{-0.2x}(1 + 5e^{-0.2x})^{-2} \)

162. Write the derivative formula for each of the following functions.

(a) \( g(t) = 2.57 - (3u^5 - 6)^4 \)
(b) \( f(t) = 5.7 + 1.4 \ln(2t) \)
(c) \( y = \sqrt{6e^x} + 2 \)
(d) \( L(x) = \frac{17.6}{1 + 3e^{-0.4x}} \)

1—4.5—ANSWER: (a) \( g'(t) = -4(3u^5 - 6)^3(15u^4) \); (b) \( f'(t) = \frac{14}{t} \); (c) \( y' = \frac{3e^x}{\sqrt{6e^x + 2}} \);
(d) \( L'(x) = 21.12e^{-0.4x}(1 + 3e^{-0.4x})^{-2} \)
Write the derivative formula for each of the following functions. Include units of measure with each answer.

(a) \( g(t) = (3t^5 - 6)^4 - 2.57 \) tickets when \( t \) students are waiting in line
(b) \( f(x) = 5.7 + 1.4 \ln(2x) \) feet when \( x \) liters of water are removed
(c) \( L(x) = \frac{17.6}{1 + 3e^{-0.4x}} \) dollars in \( x \) quarters

1—4.5—ANSWER: (a) \( g'(t) = 4(3t^5 - 6)^3(15t^4) \) tickets per student; (b) \( f'(x) = \frac{14}{x} \) feet per liter; (c) \( L'(x) = 21.12e^{-0.4x}(1 + 3e^{-0.4x})^{-2} \) dollars per quarter

At the end of an advertising campaign, the monthly sales of a new CD declined. Monthly sales are given by the equation \( S(x) = 33,815e^{-0.21x} \) dollars where \( x \) is the number of months after the advertising campaign ended.

(a) Write the rate of change formula for monthly sales of the new CD.
(b) How rapidly were monthly sales changing at the end of the campaign?
(c) What were monthly sales and how rapidly were they changing 2 months after the campaign ended?

2—4.5—ANSWER: (a) \( S'(x) = -7101.15e^{-0.21x} \) dollars per month \( x \) months after the campaign ended; (b) \( -7101.15 \) dollars per month; (c) Monthly sales were \( S(2) \approx 22,218.04; S'(2) \approx -4665.79 \) dollars per month, so monthly sales were declining by about \$4665.79\) 2 months after the campaign ended.

The dividends paid per share of Coca-Cola stock from 1987 through 1996 can be modeled by \( D(t) = 0.132e^{0.150833t} \) dollars \( t \) years after 1987. (Source: Based on data from Hoover's Company Profiles)

(a) Write the rate-of-change formula for Coca-Cola's dividends.
(b) How rapidly were Coke's dividends changing in 1987?
(c) How rapidly were Coke's dividends changing in 1996?

2—4.5—ANSWER: (a) \( D'(t) = 0.132(0.150833)e^{0.150833t} \) dollars per year \( t \) years after 1987; (b) \( D'(0) \approx 0.02 \) dollars per year; (c) \( D'(9) \approx 0.08 \) dollars per year

The amount of currency in circulation in the United States from 1945 through 1996 can be modeled by \( C(x) = 16,567.874e^{0.060539x} \) million dollars \( x \) years after 1945. (Source: Based on data from the U.S. Department of the Treasury)

(a) Write the rate-of-change formula for the amount of currency in circulation.
(b) How rapidly was the amount of currency in circulation changing in 1950?
(c) How rapidly was the amount of currency in circulation changing in 1976?
(d) How rapidly was the amount of currency in circulation changing in 1996?

2—4.5—ANSWER: (a) \( C'(x) = 16,567.874(0.060539)e^{0.060539x} \) million dollars per year \( x \) years after 1945; (b) \( C'(5) \approx 1358 \) million per year; (c) \( C'(31) \approx 6552 \) million dollars per year; (d) \( C'(51) \approx 21,988 \) million dollars per year
167. Find $f'(27)$ if $f(x) = m(x) \cdot n(x)$, $m(27) = 5$, $m'(27) = -2$, $n(27) = 11$, and $n'(27) = 9$.

1—4.6—ANSWER: $f'(27) = m(27) \cdot n'(27) + m'(27) \cdot n(27) = 5(9) + (-2)(11) = 23$

168. Find $a'(-3.6)$ if $a(x) = b(x) \cdot c(x)$, $b(-3.6) = -2.2$, $b'(-3.6) = 12.7$, $c(-3.6) = -4.8$, and $c'(-3.6) = 16.3$.

1—4.6—ANSWER: $a'(-3.6) = b(-3.6) \cdot c'(-3.6) + b'(-3.6) \cdot c(-3.6) = -2.2(16.3) + 12.7(-4.8) = -96.82$

169. Find $z'(3)$ if $z(t) = x(t) \cdot y(t)$, $x(3) = -22$, $x'(3) = -8.9$, $y(3) = 38$, and $y'(3) = 7$.

1—4.6—ANSWER: $z'(3) = x(3) \cdot y'(3) + x'(3) \cdot y(3) = -22(7) + (-8.9)(38) = -492.2$

170. Let $s$ be the total student enrollment at a certain university, and let $j$ be the proportion of total enrollment that are juniors (expressed as a decimal). For both functions, the input $t$ is the number of years since 2000.

(a) Write sentences interpreting the following mathematical statements:
   i. $s(2) = 20,100$
   ii. $s'(2) = 870$
   iii. $j(2) = 0.25$
   iv. $j'(2) = -0.01$

(b) If $H(t) = s(t) \cdot j(t)$, describe the input and output of $H$.

(c) Find the values of $H(2)$ and $H'(2)$. Interpret your answers.

2—4.6—ANSWER: (a) i. In 2002, there are 20,100 students enrolled at the university. ii. Total enrollment is increasing by 870 students per year in 2002. iii. In 2002 juniors account for 25% of the student population. iv. The proportion of total enrollment that represents juniors is decreasing by 1 percentage point per year in 2002. (b) Input: The number of years since 2000; Output: The number of juniors enrolled at the university; (c) $H(2) = s(2) \cdot j(2) = 20,100(0.25) = 5025$ juniors; $H'(2) = s(2) \cdot j'(2) + s'(2) \cdot j(2) = 20,100(-0.01) + 870(0.25) = 16.5$; In 2002 there are 5025 juniors enrolled at the university, and that number is increasing at a rate of 16.5 juniors per year.

171. Let $D$ be the demand for an airline ticket when the price of a ticket is $x$ dollars.

(a) Write sentences interpreting these mathematical statements: $D(300) = 250$, $D'(300) = -1$.

(b) Give a formula for the revenue $R$ generated from the sale of these airline tickets when the price of each ticket is $x$ dollars.

(c) Find $R'(x)$ when $x = 300$. Interpret your answer.

2—4.6—ANSWER: (a) When the price of an airline ticket is $300, 250 tickets are demanded, and the demand is decreasing at a rate of 1 ticket per dollar increase in price; (b) $R(x) = x \cdot D(x)$ dollars when tickets cost $x$ each; (c) $R'(x) = x \cdot D'(x) + 1 \cdot D(x)$, so $R'(300) = -50$ dollars. When each ticket costs $300, revenue is decreasing at a rate of $50 per dollar of ticket price.
Let \( P \) be the price charged for one calculator when \( x \) calculators are sold.
(a) Write sentences interpreting the following mathematical statements: \( P(1500) = 7.25 \) and 
\( P'(1500) = -0.002. \)
(b) Give a formula for the revenue \( R \) generated from the sale \( x \) calculators.
(c) Find the values of \( R(1500) \) and \( R'(1500) \). Interpret your answers.

2—4.6—ANSWER: (a) When 1500 calculators are sold, the price per calculator is $7.25, and 
the price is falling by $0.002 per calculator; (b) \( R(x) = x \cdot P(x) \) dollars when \( x \) calculators 
are sold; (c) \( R(1500) = 1500 \cdot P(1500) = 1500(7.25) = \$10,875. \) Now \( R'(x) = x \cdot P'(x) + 1 \cdot P(x), \) 
so \( R'(1500) = 1500(-0.002) + 7.25 = $4.25. \) When 1500 calculators are sold, the revenue that is 
generated is $10,875. At that time, the revenue is increasing by $4.25 per calculator.

The number of passengers on the ships owned by Advent Cruise Lines \( x \) years after 1995 can 
be modeled by \( p(x) = 2.6x^2 + 80x + 17,900 \) passengers. The amount spent each year on 
advertising to attract passengers on cruise ships is given by \( \alpha(x) = 34(1.06^x) \) dollars per 
passenger where \( x \) is the number of years after 1995.
(a) What are the input and output units of measure for the function \( C(x) = p(x) \cdot \alpha(x)? \)
(b) Write a formula in terms of \( x \) for the derivative of \( C. \)
(c) Find the value of and write a sentence interpreting each of the following:
   i. \( p(4) \)
   ii. \( \alpha(4) \)
   iii. \( C'(4) \)

2—4.6—ANSWER: (a) Input units: years (after 1995); Output units: dollars; (b) \( C'(x) = 
p(x) \cdot \alpha'(x) + p'(x) \cdot \alpha(x) = (2.6x^2 + 80x + 17,900)(34)(1.06^x) + (5.2x + 80)(34)(1.06^x) \) 
dollars per year; (c) i. \( p(4) \approx 18,262. \) In 1999 there were about 18,262 passengers who sailed 
on Advent Cruise ships. ii. \( \alpha(4) \approx 42.92; \) About $42.92 was spent on each passenger in 1999 
to attract passengers on cruise ships. iii. \( C'(4) \approx 50,001.71; \) The total amount spent on 
advertising to attract the passengers who sailed on Advent Cruise ships in 1999 was about 
$50,002.

The number of households in a certain community is given by \( h(x) = 1.26x^2 + 7800 \) households 
after \( x \) years. The proportion (expressed as a decimal) of the households in this same community 
that have children under the age of 18 is given by \( p(x) = 0.0031x + 0.22 \) after \( x \) years.
(a) What does \( C(x) = h(x) \cdot p(x) \) represent?
(b) Give a formula for \( C'(x). \)
(c) Find and interpret the following:
   i. \( h(3) \) and \( h'(3) \)
   ii. \( p(3) \) and \( p'(3) \)
   iii. \( C(3) \) and \( C'(3) \)

2—4.6—ANSWER: (a) \( C \) gives the number of households in the community that have 
children under the age of 18. (b) \( C'(x) = (1.26x^2 + 7800)(0.0031) + (2.52x)(0.0031x + 0.22) \) 
households per year; (c) i. \( h(3) = 7811.34, h'(3) = 7.56; \) After 3 years, there are about 7811 
households in the community and this number is increasing by 7.56 households per year; ii. 
\( p(3) = 0.2293, p'(3) = 0.0031; \) After 3 years, 22.93% of households have children under the 
age of 18 and this percentage is increasing by 0.31 percentage points per year; iii. \( C(3) = 
h(3) \cdot p(3) \approx 1791 \) households with children under age 18; \( C'(3) = h(3) \cdot p'(3) + h'(3) \cdot p(3) \approx 
26; \) After 3 years, there are about 1791 households with children under age 18, and that 
number is growing at a rate of about 26 households per year.
Find the derivative formula for each of the following functions:
(a) \( f(x) = (4x^3 - 9x)(7x^2 - 2x + 5) \)
(b) \( h(x) = 6x^3 e^{2x + 3} \)

1—4.6—ANSWER: (a) \( f'(x) = (4x^3 - 9x)(14x - 2) + (7x^2 - 2x + 5)(12x^2 - 9) \)
(b) \( h'(x) = 18x^2 e^{2x + 3} + 12x^3 e^{2x + 3} \)

Find the derivative formula for each of the following functions:
(a) \( h(t) = (t^2 - 9)(t^3 - 3t^2 + 2t - 1)^2 \)
(b) \( r(x) = 3x^5 \ln(x^3) \)

1—4.6—ANSWER: (a) \( h'(t) = (t^2 - 9)(2t^3 - 3t^2 + 2t - 1)(3t^2 - 6t + 2) + 2t(t^3 - 3t^2 + 2t - 1)^2 \)
(b) \( r'(x) = (3x^5)(3x^{-1}) + (15x^4) \ln(x^3) \)

Write the derivative formula for each of the following functions:
(a) \( P(t) = (15t + 7)^2 (9t^2 - 4)^4 \)
(b) \( W(x) = e^{2x} (14x - 3) \)

1—4.6—ANSWER: (a) \( P'(t) = (15t + 7)^2 (4)(9t^2 - 4)^3 (18t) + 2(15t + 7)(30)(9t^2 - 4)^4 \)
(b) \( W'(x) = 14e^{2x} + 2e^{2x} (14x - 3) \)

Give the derivative formula for each of the following functions:
(a) \( h(t) = (5t)(9t + 2)^3 \)
(b) \( g(x) = (1.25^x) \ln(3x) \)

2—4.6—ANSWER: (a) \( h'(t) = (\ln 5)(5t)(9t + 2)^3 + 27(5t)(9t + 2)^2 \)
(b) \( g'(x) = (\ln 1.25)(1.25^x) \ln(3x) + (1.25^x)(x^{-1}) \)

Give the derivative formula for each of the following functions:
(a) \( f(x) = 100(1.075^x)(3x^2 + 8x + 4.4) \)
(b) \( g(x) = (1 - 3x - 16x^2)(14x - 9)^{-2} \)

2—4.6—ANSWER: (a) \( f'(x) = 100(1.075^x)(6x + 8) + 100(\ln 1.075)(1.075^x)(3x^2 + 8x + 4.4) \)
(b) \( g'(x) = (1 - 3x - 16x^2)(-2)(14x - 9)^{-3} + (-3 - 32x)(14x - 9)^{-2} \)

Give the derivative formula for the function \( f(x) = \frac{2.05^x}{1 - 3x} \).

2—4.6—ANSWER: \( f'(x) = (\ln 2.05)(2.05^x)(1 - 3x)^{-1} + (2.05^x)(-1)(1 - 3x)^{-2}(-3) \)