Exam 1 will be on 09/25/12 and covers the following sections: 1.5, 1.6, 2.1, 2.2, 2.3, 2.4, 2.5.

SHORT ANSWER. Write the word or phrase that best completes each statement or answers the question.

The graph of \( y = f(x) \) is shown. Use the graph to answer the question.

1) Find \( \lim_{x \to (-1)^-} f(x) \) and \( \lim_{x \to (-1)^+} f(x) \).

2) Find \( \lim_{x \to -1} f(x) \) and \( f(-1) \).

3) Find \( \lim_{x \to 0} f(x) \) and \( f(0) \).
4) Find \( \lim_{x \to 0^-} f(x) \) and \( \lim_{x \to 0^+} f(x) \).

5) Find \( \lim_{x \to 0} f(x) \) and \( f(0) \).

6) Find \( \lim_{x \to 1} f(x) \) and \( f(1) \).
7) Find \( \lim_{{x \to 0^+}} f(x) \) and \( \lim_{{x \to 0^-}} f(x) \).

8) Find \( \lim_{{x \to -1^-}} f(x) \) and \( f(-1) \).

9) Find \( \lim_{{x \to -1^+}} f(x) \) and \( \lim_{{x \to -1^-}} f(x) \).
10) \( f(x) = \begin{cases} 
2, & \text{for } x = 1, \\
2.5, & \text{for } x \neq 1 
\end{cases} \)

Find \( \lim_{x \to 1} f(x) \) and \( f(1) \).

11) \( f(x) = \begin{cases} 
2, & \text{for } x = 1, \\
2.5, & \text{for } x \neq 1 
\end{cases} \)

Find \( \lim_{x \to 1^+} f(x) \) and \( \lim_{x \to 1^-} f(x) \).
12) \( f(x) = \begin{cases} 
4, & \text{for } x = 0, \\
\frac{1}{2}x^2 - 1.5, & \text{for } x \neq 0 
\end{cases} \)

Find \( \lim_{x \to 0^+} f(x) \) and \( \lim_{x \to 0^-} f(x) \).

13) \( f(x) = \begin{cases} 
2, & \text{for } x = 0, \\
\frac{1}{2}x^2 - 1.5, & \text{for } x \neq 0 
\end{cases} \)

Find \( \lim_{x \to 0} f(x) \) and \( f(0) \).

Find the limit.

14) \( \lim_{x \to 3} \frac{5x + 2}{x + 3} \)

15) \( \lim_{x \to -2} -8x^2 + 3x - 8 \)

16) \( \lim_{x \to -2} (3x^5 - 3x^4 + 4x^3 + x^2 + 5) \)
17) \( \lim_{x \to 1} \sqrt{x^2 + 8x + 16} \)

18) \( \lim_{x \to 3} \frac{x^2 + 9}{x + 3} \)

19) \( \lim_{x \to -4} \frac{5x - 7}{6x^2 + 8x - 3} \)

Decide whether the limit exists. If it exists, find its value.

20) \( \lim_{x \to \infty} f(x) \)

21) \( \lim_{x \to 2} (3x^5 - 3x^4 - 4x^3 + x^2 - 5) \)

22) \( \lim_{x \to 7} \sqrt{x^2 + 8x + 16} \)

Find the limit if it exists.

23) \( \lim_{x \to \frac{3}{7}} \frac{7x(x - \frac{1}{2})}{\frac{7}{2}} \)

Find the limit if it exists.

24) \( \lim_{x \to 5} \frac{x^2 - 2x - 15}{x + 3} \)

25) \( \lim_{x \to 2} \frac{x^2 - 4}{x - 2} \)
26) \( \lim_{x \to -3} \frac{x^2 + 6x + 9}{x + 3} \)

27) \( \lim_{x \to 5} \frac{x^2 + 2x - 35}{x - 5} \)

28) \( \lim_{x \to 1} \frac{x^2 - 1}{x^2 - 3x + 2} \)

29) \( \lim_{x \to -6} \frac{x^2 + 8x + 12}{x^2 + 5x - 6} \)

Use the properties of limits to help decide whether the limit exists. If the limit exists, find its value.

30) \( \lim_{x \to 1} \frac{x^2 + 4x - 5}{x - 1} \)

31) \( \lim_{x \to 4} \frac{x^2 + 4x - 32}{x^2 - 16} \)

Find the limit, if it exists.

32) \( \lim_{x \to \infty} \frac{x^2 - 7x + 13}{x^3 - 4x^2 + 7} \)

33) \( \lim_{x \to -\infty} \frac{-2x^2 - 9x + 2}{-3x^2 - 6x + 18} \)

34) \( \lim_{x \to \infty} \frac{5x + 1}{11x - 7} \)

35) \( \lim_{x \to \infty} \frac{10x^3 - 3x^2 + 3x}{-x^3 - 2x + 5} \)

36) \( \lim_{x \to -\infty} \frac{5x^3 + 4x^2}{x - 7x^2} \)

Evaluate the limit if one exists.

37) \( \lim_{x \to \infty} \frac{2x^2 + 7}{x^2 - 2x} \)

38) \( \lim_{x \to \infty} \frac{x^2 - 7x + 8}{x^3 + 6x^2 + 9} \)
Find the derivative by using the definition of the derivative.

39) \( f(x) = 4x^2 \)

40) \( f(x) = -9x^2 \)

41) \( f(x) = x^2 - 9x \)

42) \( f(x) = 2x^2 + 1 \)

43) \( f(x) = 2x^2 - 9x \)

44) \( f(x) = 5x^2 + 9x + 7 \)

Find \( \frac{dy}{dx} \).

45) \( y = x^6 \)

46) \( y = 5x^2 + 6x + 8 \)

47) \( y = 2x^4 + 3x^3 - 7 \)

48) \( y = \frac{4}{x} - \frac{x}{8} \)

49) \( y = 16x^2 - 2x^3 + 4x \)

50) \( y = \frac{9}{x^5} - \frac{3}{x} \)

51) \( y = (5x + 2)^2 \)

52) \( y = \frac{3 + 9x}{x} \)

53) \( y = \frac{3 + 6x - 3x^2}{x} \)

54) \( y = \frac{5 - 7x + 6x^2}{x^2} \)

55) \( y = 4x^4 + 4x^3 + 7 \)

56) \( y = \frac{7}{x} - \frac{x}{2} \)
57) \( y = 17x^2 - 5x^3 + 12x \)

58) \( y = \frac{3}{x^3} - \frac{3}{x} \)

59) \( y = -8\sqrt{x} \)

60) \( y = \sqrt[3]{x^8} \)

61) \( y = \frac{x + 7}{\sqrt{x}} \)

62) \( y = 4x^{1/4} - 3 \)

63) \( y = x^{7/6} + 4x^{-3/4} \)

64) \( y = 8x^{-1/3} - 6x^{1/6} \)

65) \( y = \frac{x + 9}{\sqrt{x}} \)

66) \( y = 7x^{1/7} - 9 \)

67) \( y = x^{4/3} + 6x^{-5/6} \)

68) \( y = 5x^{-1/2} - 5x^{1/5} \)

Solve the problem.

69) The total cost to produce \( x \) handcrafted wagons is \( C(x) = 130 + 7x - x^2 + 7x^3 \). Find the rate of change of cost with respect to the number of wagons produced (the marginal cost) when \( x = 5 \).

70) The volume of a sphere in terms of its radius, \( r \), is given by \( V = \frac{4}{3}\pi r^3 \). Find the rate of change of the volume with respect to the radius when the radius is 12 cm. Use 3.14 for \( \pi \) and round your answer to the nearest tenth.

71) \( A(x) = -0.015x^3 + 1.05x \) gives the alcohol level in an average person's bloodstream \( x \) hours after drinking 8 oz of 100-proof whisky. If the level exceeds 1.5 units, a person is legally drunk. Find the rate of change of alcohol level with respect to time when \( x = 2 \) hours.
72) The median weight, \( w \), of a girl between the ages of 0 and 36 months can be approximated by the function
\[
w(t) = 0.0006t^3 - 0.0484t^2 + 1.61t + 7.60,
\]
where \( t \) is measured in months and \( w \) is measured in pounds.
For a girl of median weight, find the rate of change of weight with respect to time at age 20 months.

73) For a motorcycle traveling at speed \( v \) (in mph) when the brakes are applied, the distance \( d \) (in feet) required to stop the motorcycle may be approximated by the formula
\[
d = 0.05v^2 + v.
\]
Find the instantaneous rate of change of stopping distance with respect to speed when the speed is 41 mph.

74) The power \( P \) (in W) generated by a particular windmill is given by \( P = 0.015V^3 \) where \( V \) is the velocity of the wind (in mph). Find the instantaneous rate of change of power with respect to velocity when the velocity is 11.3 mph. Round to the nearest tenth.

75) A ball is thrown vertically upward from the ground at a velocity of 106 feet per second. Its distance from the ground after \( t \) seconds is given by \( s(t) = -16t^2 + 106t \). How fast is the ball moving 5 seconds after being thrown?

**Differentiate.**

76) \( f(x) = (4x + 2)^2 \)

77) \( f(x) = (2x - 4)(\sqrt{x} + 4) \)

78) \( f(x) = \left( x + \frac{3}{x} \right)(x^2 - 4) \)

[Do not use algebra before differentiating]

79) \( (5x^5 + 3x^3 - 1)(9x^2 - 7\sqrt{x}) \)

[Do not use algebra before differentiating]

80) \( f(x) = \left( x + \frac{3}{x} \right)(x^2 - 4) \)

[Do not use algebra before differentiating]

81) \( g(x) = \frac{x^2}{x - 11} \)

82) \( h(r) = \frac{r^2 + 2r - 8}{3r - 7} \)

83) \( q(t) = \frac{5t}{t^2 - 5t - 4} \)

84) \( q(t) = \frac{t^2 - 3t + 3}{t^2 + 4t - 7} \)
85) \( y = \frac{\sqrt{x + 6}}{\sqrt{x - 6}} \)

86) \( f(x) = \frac{(x - 7)(x^2 + 2x)}{x^3} \)

87) \( f(x) = \left( \frac{1 + 2x}{2x} \right)(2 - x) \)

88) \( f(t) = \left( \frac{t^6 + 4}{2t} \right) \left( \frac{t^8 + 6}{t} \right) \)

89) \( f(x) = \frac{(2x - 1)(3x^2 + 1)}{3x + 2} \)

90) \( f(x) = \frac{(x - 6)(x^2 + 3x)}{x^3} \)

91) \( f(x) = \left( \frac{1 + 9x}{9x} \right)(9 - x) \)

92) \( f(t) = \left( \frac{t^7 + 4}{2t} \right) \left( \frac{t^8 + 6}{t} \right) \)

Solve the problem.

93) Murrel’s formula for calculating the total amount of rest, in minutes, required after performing a particular type of work activity for 30 minutes is given by the formula \( R(w) = \frac{30(w - 4)}{w - 1.5} \), where \( w \) is the work expended in kilocalories per min. A bicyclist expends 7 kcal/min as she cycles home from work. Find \( R'(w) \) for the cyclist; that is, find \( R'(7) \). Round to the nearest hundredth.

94) Prairie dogs form an important part of the coyote’s diet. As coyotes are hunting for prairie dogs, they must be careful to expend just the right amount of time at each burrow. If a coyote spends too little time at each burrow, it catches very few prairie dogs per kilocalorie of energy expended. Likewise, if the coyote spends too much time digging at a single burrow, it can expend a large amount of energy per prairie dog caught. The relation between energy expended and time spent at each burrow is approximated by \( E = \left( \frac{1}{t} + \frac{20}{t - 0.75} \right)^2 \) for \( t > 0.75 \) minutes, where \( t \) is in minutes and \( E \) is in kcal expended per prairie dog caught. How much time should a coyote spend at each burrow to minimize the energy expended per prairie dog caught. (Hint: pay close attention to the domain of the above function.)

Differentiate.

95) \( y = (3 - 8x)^{260} \)
96) \( y = (2x^2 + 5)^5 \)

97) \( y = \sqrt[3]{8x^2 - x} \)

98) \( y = \frac{1}{(2x^2 + 7x + 7)^4} \)

99) \( g(x) = \left[ 6x^3 - 9x + \frac{1}{x^2} \right]^{7/5} \)

100) \( y = (4x^2 - 9)^4 - (1 + 4x^3)^5 \)

101) \( f(x) = (4x^5 - 4x^4 + 3)^{308} \)

102) \( f(x) = 3x(4x + 2)^4 \)

103) \( f(x) = (6x + 3)\sqrt{2x - 5} \)

104) \( y = \frac{4x + 3}{\sqrt{x^2 - 2}} \)

105) \( f(x) = \left( \frac{3x + 5}{x - 5} \right)^5 \)

106) \( h(z) = \frac{4}{\sqrt[4]{8z + 9}} \)

**Solve the problem.**

107) $1000 is deposited in an account with an interest rate of \( r\% \) per year, compounded monthly. At the end of 8 years, the balance in the account is given by

\[ A = 1000 \left( 1 + \frac{r}{1200} \right)^96 \]

Find the rate of change of \( A \) with respect to \( r \) when \( r = 10 \). Round your answer to the nearest hundredth, if necessary.

108) If $2000 is invested at interest rate \( i \), compounded quarterly, it will grow in 3 years to an amount \( A \), in $, given by:

\[ A = 2000 \left( 1 + \frac{i}{4} \right)^{12} \]

Find the rate of change, \( \frac{dA}{di} \).
Find the second derivative.

109) \( y = 7x^4 - 7x^2 + 8 \)

110) \( y = (3x + 5)^2 \)

111) \( y = 5x^4 - 7x^2 + 3 \)

112) \( y = (3x + 5)^2 \)

113) \( y = (x^2 + 3x)^{40} \)

114) \( y = \sqrt{3x - 7} \)

115) \( f(x) = \frac{3}{\sqrt{4x + 5}} \)

Find the indicated derivative of the function.

116) \( \frac{d^4y}{dx^4} \) of \( y = 6x^5 - 4x^2 - 3x + 1 \)

117) \( \frac{d^4y}{dx^4} \) of \( y = 3x^6 - 4x^4 + 5x^2 \)

118) \( \frac{d^5y}{dx^5} \) of \( y = 4x^6 + 3x^4 + 5x^2 + 4 \)

119) \( \frac{d^6y}{dx^6} \) of \( y = 4x^7 - 3x^5 - 2x^3 - 6 \)

Solve the problem.

120) A company estimates that the daily revenue (in dollars) from the sale of \( x \) cookies is given by \( R(x) = 1705 + 0.05x + 0.0003x^2 \). Currently, the company sells 480 cookies per day. Use marginal revenue to estimate the increase in revenue if the company increases sales by one cookie per day.

121) A grocery store estimates that the weekly profit (in dollars) from the production and sale of \( x \) cases of soup is given by \( P(x) = -5700 + 9.8x - 0.0015x^2 \) and currently 1200 cases are produced and sold per week. Use the marginal profit to estimate the increase in profit if the store produces and sells one additional case of soup per week.
122) A company estimates that the daily cost (in dollars) of producing x chocolate bars is given by \( C(x) = 1435 + 0.05x + 0.0003x^2 \). Currently, the company produces 860 chocolate bars per day. Use marginal cost to estimate the increase in the daily cost if one additional chocolate bar is produced per day.

123) The weekly profit, in dollars, from the production and sale of x bicycles is given by \( P(x) = 10.00x - 0.005x^2 \). Currently, the company produces and sells 1000 bicycles per week. Use the marginal profit to estimate the change in profit if the company produces and sells one more bicycle per week.

124) A company finds that when it spends x million dollars on advertising, its profit \( P \), in thousands of dollars, is given by \( P(x) = 770 + 20x - 2x^2 \). Currently, the company spends 14 million dollars on advertising. Use the marginal profit to estimate the change in profit if the company increases its advertising expenditure by one million dollars.

125) Suppose that the daily cost, in dollars, of producing x televisions is \( C(x) = 0.002x^3 + 0.1x^2 + 74x + 520 \), and currently 40 televisions are produced daily. Use \( C(40) \) and the marginal cost to estimate the daily cost of increasing production to 43 televisions daily. Round to the nearest dollar.

126) Suppose that the weekly profit, in dollars, of producing and selling x cars is \( P(x) = -0.005x^3 - 0.3x^2 + 880x - 1000 \), and currently 40 cars are produced and sold weekly. Use \( P(40) \) and the marginal profit when \( x = 40(P'(40)) \) to estimate the weekly profit of producing and selling 41 cars. Round to the nearest dollar.

127) A supply function for a certain product is given by \( S(p) = 0.06p^3 + 4p^2 + 11p - 13 \), where \( S(p) \) is the number of items produced when the price is \( p \) dollars. Use \( S'(p) \) to estimate how many more units a producer will supply when the price changes from $14.00 per unit to $14.10 per unit.

128) Suppose the demand for a certain item is given by \( D(p) = -3p^2 + 5p + 8 \), where \( p \) represents the price of the item. Find \( D'(p) \), the rate of change of demand with respect to price.

129) The diameter of a circle is given by the formula \( D = \frac{C}{\pi} \), where \( C \) is the circumference. The diameter of a tree was 8 in. During the following year, the circumference increased by 2 in. Use \( D'(C) \) to estimate how much the tree’s diameter increased in that year.

130) The volume of a sphere is given by the formula \( V = \frac{4}{3}\pi r^3 \) where \( r \) is the radius. A tumor is approximately spherical in shape. Use \( V'(r) \) to estimate the increase in volume of the tumor if its radius increases from 7 mm to 10 mm. Round to the nearest 100 mm\(^3\).
Answer Key
Testname: MAC_2233_FALL_12_EXAM_1_REVIEW

1) -2, -7
2) does not exist, -7
3) 0, does not exist
4) 1, -1
5) does not exist, 4
6) does not exist, does not exist
7) -0.5, -0.5
8) does not exist, -1
9) -1, -2
10) 2.5, 2
11) 2.5, 2.5
12) -1.5, -1.5
13) -1.5, 2
14) 17
15) -46
16) -167
17) 5
18) 3
19) $\frac{27}{61}$
20) \(\infty\)
21) 15
22) 11
23) $-\frac{3}{14}$
24) 0
25) 4
26) 0
27) 12
28) -2
29) $\frac{4}{7}$
30) 6
31) $\frac{3}{2}$
32) 0
33) $\frac{2}{3}$
34) $\frac{5}{11}$
35) -10
36) \(\infty\)
37) 2
38) 0
39) 8x
40) -18x
41) 2x - 9
42) 4x
43) 4x - 9
Answer Key
Testname: MAC_2233_FALL_12_EXAM_1_REVIEW

44) 10x + 9
45) \( \frac{dy}{dx} = 6x^5 \)
46) \( \frac{dy}{dx} = 10x + 6 \)
47) \( \frac{dy}{dx} = 8x^3 + 9x^2 \)
48) \( \frac{dy}{dx} = -\frac{4}{x^2} - \frac{1}{8} \)
49) \( \frac{dy}{dx} = -32x^{-3} - 6x^2 + 4 \)
50) \( \frac{dy}{dx} = -\frac{45}{x^6} + \frac{3}{x^2} \)
51) \( \frac{dy}{dx} = 50x + 20 \)
52) \( \frac{dy}{dx} = -\frac{3}{x^2} \)
53) \( \frac{dy}{dx} = -\frac{3}{x^2} - 3 \)
54) \( \frac{dy}{dx} = -\frac{10}{x^3} + \frac{7}{x^2} \)
55) \( \frac{dy}{dx} = 16x^3 + 12x^2 \)
56) \( \frac{dy}{dx} = -\frac{7}{x^2} - \frac{1}{2} \)
57) \( \frac{dy}{dx} = -34x^{-3} - 15x^2 + 12 \)
58) \( \frac{dy}{dx} = -\frac{9}{x^4} + \frac{3}{x^2} \)
59) \( \frac{dy}{dx} = -\frac{4}{\sqrt{x}} \)
60) \( \frac{dy}{dx} = \frac{8}{9\sqrt{x}} \)
61) \( \frac{1}{2\sqrt{x}} - \frac{7}{2x^{3/2}} \)
62) \( \frac{dy}{dx} = \frac{1}{x^{3/4}} \)
63) \( \frac{dy}{dx} = \frac{7\sqrt{x}}{6} - \frac{3}{x^{7/4}} \)
64) \( \frac{dy}{dx} = -\frac{8}{3x^{4/3}} - \frac{1}{x^{5/6}} \)
65) $\frac{1}{2\sqrt{x}} - \frac{9}{2x^{3/2}}$

66) $\frac{dy}{dx} = \frac{1}{x^{6/7}}$

67) $\frac{dy}{dx} = \frac{4\sqrt[3]{x}}{3} - \frac{5}{x^{11/6}}$

68) $\frac{dy}{dx} = -\frac{5}{2x^{3/2}} - \frac{1}{x^{4/5}}$

69) $522$ per wagon

70) $1808.6$ cm$^3$/cm

71) $0.87$ units/hr

72) $0.394$ lb/mo

73) $5.1$ ft/mph

74) $5.7$ W/mph

75) $-54$ ft per sec

76) $32x + 16$

77) $3x^{1/2} - 2x^{-1/2} + 8$

78) $2x\left(x + \frac{3}{x}\right) + \left(1 - \frac{3}{x^2}\right)(x^2 - 4)$

79) $(9x^2 - 7\sqrt{x})(25x^4 + 9x^2) + \left(18x - \frac{7}{2\sqrt{x}}\right)(5x^5 + 3x^3 - 1)$

80) $2x\left(x + \frac{3}{x}\right) + \left(1 - \frac{3}{x^2}\right)(x^2 - 4)$

81) $\frac{x^2 - 22x}{(x - 11)^2}$

82) $\frac{3r^2 - 14r + 10}{(3r - 7)^2}$

83) $\frac{-5(t^2 + 4)}{(t^2 - 5t - 4)^2}$

84) $\frac{7t^2 - 20t + 9}{(t^2 + 4t - 7)^2}$

85) $-\frac{6}{\sqrt{x}(\sqrt{x} - 6)^2}$

86) $f'(x) = \frac{5}{x^2} + \frac{28}{x^3}$

87) $f'(x) = -\frac{1}{x^2} - 1$

88) $f'(x) = 6t^{11} + 12t^5 + 12t^3 - \frac{24}{t^3}$

89) $f'(x) = \frac{36x^3 + 27x^2 - 12x + 7}{(3x + 2)^2}$
90) \( f'(x) = \frac{3}{x^2} + \frac{36}{x^3} \)

91) \( f'(x) = -\frac{1}{x^2} - 1 \)

92) \( f'(t) = \frac{13}{2}t^2 + 12t^3 + 15t^4 - \frac{24}{t^3} \)

93) 2.48 min\(^2$/kcal

94) 1.5 min

95) \(-2080(3 - 8x)^{259}\)

96) \(20x(2x^2 + 5)^4\)

97) \(\frac{16x - 1}{3(8x^2 - x)^{2/3}}\)

98) \(-\frac{4(4x + 7)}{(2x^2 + 7x + 7)^5}\)

99) \(\frac{7}{6}x^3 - 9x + \frac{1}{x^2}\) \(\left(\frac{2/5}{18x^2 - 9 - \frac{2}{x^3}}\right)\)

100) \(32x(4x^2 - 9)^3 - 60x^2(1 + 4x)^4\)

101) \(308(4x^5 - 4x^4 + 3)^3(20x^4 - 16x^3)\)

102) \(3(4x + 2)^3(20x + 2)\)

103) \(\frac{6x + 3}{\sqrt{2x - 5}} + 6\sqrt{2x - 5}\)

104) \(-\frac{3x - 8}{(x^2 - 2)^{3/2}}\)

105) \(\frac{3x + 5)^4}{x - 5} \cdot \frac{-100}{(x - 5)^2}\)

106) \(\frac{1}{4}\left(\frac{137}{(-9z + 7)^2}\right)\)

107) \(\frac{dA}{dr} = 175.99\)

108) \(\frac{dA}{di} = 6000\left(1 + \frac{i^{11}}{4}\right)\)

109) \(84x^2 - 14\)

110) 18

111) \(60x^2 - 14\)

112) 18

113) \(40(x^2 + 3x)^3(158x^2 + 474x + 351)\)

114) \(-\frac{9}{4(3x - 7)^{3/2}}\)

115) \(-\frac{32}{9(4x + 5)^{5/3}}\)

116) 720x

117) \(1080x^2 - 96\)

118) 2880x
Answer Key
Testname: MAC_2233_FALL_12_EXAM_1_REVIEW

119) 20,160x
120) $0.34
121) $6.20
122) $0.57
123) 0.00 dollars
124) -36 thousand dollars
125) $4043
126) $34,232
127) 16
128) D'((p) = -6p + 5
129) \(\frac{2}{\pi}\) in.
130) 1800 mm\(^3\)