Exam 2 will be on 10/11/12 and covers the following sections: 2.5, 2.6, 3.1, 3.2, 3.3, 3.4, 3.5.

SHORT ANSWER. Write the word or phrase that best completes each statement or answers the question.

Solve the problem.

1) A company estimates that the daily revenue (in dollars) from the sale of $x$ cookies is given by $R(x) = 1385 + 0.05x + 0.0006x^2$.
Currently, the company sells 360 cookies per day. Use marginal revenue to estimate the increase in revenue if the company increases sales by one cookie per day.

2) A grocery store estimates that the weekly profit (in dollars) from the production and sale of $x$ cases of soup is given by $P(x) = -5600 + 9.6x - 0.0015x^2$
and currently 1000 cases are produced and sold per week. Use the marginal profit to estimate the increase in profit if the store produces and sells one additional case of soup per week.

3) A company estimates that the daily cost (in dollars) of producing $x$ chocolate bars is given by $C(x) = 880 + 0.04x + 0.0006x^2$.
Currently, the company produces 830 chocolate bars per day. Use marginal cost to estimate the increase in the daily cost if one additional chocolate bar is produced per day.

4) The weekly profit, in dollars, from the production and sale of $x$ bicycles is given by $P(x) = 40.00x - 0.005x^2$
Currently, the company produces and sells 1000 bicycles per week. Use the marginal profit to estimate the change in profit if the company produces and sells one more bicycle per week.

5) A company finds that when it spends $x$ million dollars on advertising, its profit $P$, in thousands of dollars, is given by $P(x) = 1020 + 30x - 3x^2$
Currently the company spends 15 million dollars on advertising. Use the marginal profit to estimate the change in profit if the company increases its advertising expenditure by one million dollars.

6) Suppose that the daily cost, in dollars, of producing $x$ televisions is $C(x) = 0.003x^3 + 0.1x^2 + 64x + 540$,
and currently 50 televisions are produced daily. Use $C(50)$ and the marginal cost to estimate the daily cost of increasing production to 52 televisions daily. Round to the nearest dollar.

7) Suppose that the weekly profit, in dollars, of producing and selling $x$ cars is $P(x) = -0.006x^3 - 0.3x^2 + 940x - 900$,
and currently 70 cars are produced and sold weekly. Use $P(70)$ and the marginal profit when $x = 70(P'(70))$ to estimate the weekly profit of producing and selling 73 cars. Round to the nearest dollar.
8) A supply function for a certain product is given by
\[ S(p) = 0.05p^3 + 5p^2 + 9p - 3, \]
where \( S(p) \) is the number of items produced when the price is \( p \) dollars. Use \( S'(p) \) to estimate how many more units a producer will supply when the price changes from $13.00 per unit to $13.50 per unit.

9) Suppose the demand for a certain item is given by \( D(p) = -4p^2 + 8p + 4 \), where \( p \) represents the price of the item. Find \( D'(p) \), the rate of change of demand with respect to price.

10) The diameter of a circle is given by the formula \( D = \frac{C}{\pi} \) where \( C \) is the circumference. The diameter of a tree was 11 in. During the following year, the circumference increased by 2 in. Use \( D'(C) \) to estimate how much the tree's diameter increased in that year.

11) The volume of a sphere is given by the formula \( V = \frac{4}{3}\pi r^3 \) where \( r \) is the radius. A tumor is approximately spherical in shape. Use \( V'(r) \) to estimate the increase in volume of the tumor if its radius increases from 7 mm to 9 mm. Round to the nearest 100 mm\(^3\).

Find \( dy/dx \) by implicit differentiation.

12) \( 8y^2 + 9x^2 = 17 \)

13) \( \frac{1}{3}x^3 - 3y^2 = 1 \)

14) \( 2y - x + xy = 4 \)

15) \( y^2 - xy + x^2 = 7 \)

16) \( y^2 - x^2 = 9 \)

17) \( -9xy + 7y - 2 = 0 \)

18) \( 7x^3 - x^2y^3 = 7 \)

19) \( x^3 + 3x^2y + y^3 = 8 \)

20) \( xy + x + y - x^2y^2 = 0 \)

21) \( x^{4/3} + y^{4/3} = 1 \)

22) \( \frac{x + y}{x - y} = x^2 + y^2 \)
23) \( y \sqrt{x + 1} = 4 \)

**Solve the problem.**

24) A company knows that unit cost \( C \) and unit revenue \( R \) from the production and sale of \( x \) units are related by \( C = \frac{R^2}{154,000} + 9172 \). Find the rate of change of revenue per unit when the cost per unit is changing by \( $13 \) and the revenue is \( $2000 \).

25) A product sells by word of mouth. The company that produces the product has noticed that revenue from sales is given by \( R(t) = 2\sqrt{x} \), where \( x \) is the number of units produced and sold. If the revenue keeps changing at a rate of \( $1000 \) per month, how fast is the rate of sales changing when 700 units have been made and sold? (Round to the nearest dollar per month.)

26) Water is discharged from a pipeline at a velocity \( v \) given by \( v = 1190p^{(1/2)} \), where \( p \) is the pressure (in psi). If the water pressure is changing at a rate of 0.403 psi/second, find the acceleration \( \frac{dv}{dt} \) of the water when \( p = 56 \) psi.

27) A heart attack victim is given a blood vessel dilator to increase the radii of the blood vessels. After receiving the dilator, the radii of the affected blood vessels increase at about 1% per minute. According to Poiseulle’s law, the volume of blood flowing through a vessel and the radius of the vessel are related by the formula \( V = kr^4 \) where \( k \) is a constant. What will be the percentage rate of increase in the blood flow after the dilator is given?

28) Given the revenue and cost functions \( R = 30x - 0.4x^2 \) and \( C = 3x + 15 \), where \( x \) is the daily production, find the rate of change of profit with respect to time when \( x = 10 \) units and \( \frac{dx}{dt} = 7 \) units per day.

29) Electrical systems are governed by Ohm’s law, which states the \( V = IR \), where \( V \) = voltage, \( I \) = current, and \( R \) = resistance. If the current in an electrical system is decreasing at a rate of 3 A/s while the voltage remains constant at 30 V, at what rate is the resistance increasing when the current is 40 A?

**Find the relative extrema of the function, if they exist.**

30) \( f(x) = x^2 - 10x + 35 \)

31) \( f(x) = -12x^2 - 2x - 6 \)

32) \( f(x) = x^3 - 3x^2 + 1 \)

33) \( y = x^3 - 3x^2 + 5x - 6 \)

34) \( f(x) = x^3 - 12x - 5 \)

35) \( f(x) = \frac{2}{3}x^3 - 2x^2 - 6x + 2 \)
36) \( f(x) = 3x^4 + 16x^3 + 24x^2 + 32 \)  
37) \( f(x) = x^3 - 3x^4 \)  
38) \( f(x) = \frac{x^2 + 1}{x^2} \)  
39) \( f(x) = \frac{2}{x^2 - 1} \)  
40) \( f(x) = \frac{-3}{x^2 + 1} \)  
41) \( f(x) = \frac{6x}{x^2 + 1} \)  
42) \( f(x) = \frac{x + 1}{x^2 + 2x + 2} \)  
43) \( f(x) = x^{2/5} - 1 \)  
44) \( f(x) = (x + 1)^{1/3} \)  
45) \( f(x) = (x + 5)^{2/3} - 2 \)  
46) \( f(x) = \sqrt{x^2 + 12x + 72} \)

Solve the problem.

47) A firm estimates that it will sell \( N \) units of a product after spending \( x \) dollars on advertising, where 
\[ N(x) = -x^2 + 450x + 12, \quad 0 \leq x \leq 450, \] 
and \( x \) is in thousands of dollars. Find the relative extrema of the function.

48) Assume that the temperature of a person during an illness is given by 
\[ T(t) = -0.1t^2 + 1.3t + 98.6, \quad 0 \leq t \leq 13, \] 
where \( T \) = the temperature (°F) at time \( t \), in days. Find the relative extrema of the function.

49) The Olympic flame at the 1992 Summer Olympics was lit by a flaming arrow. As the arrow moved \( d \) feet horizontally from the archer, assume that its height \( h \), in feet, was approximated by the function 
\[ h = -0.002d^2 + 0.7d + 6.7. \] 
Find the relative maximum of the function.
Find the points of inflection.

50) \( f(x) = 3x^3 + 2x + 6 \)
51) \( f(x) = x^3 + 7x + 1 \)
52) \( f(x) = -x^3 + 5x + 1 \)
53) \( f(x) = 9x - x^3 \)
54) \( f(x) = x^3 - 3x^2 + 2x + 15 \)
55) \( f(x) = x^3 + 12x^2 - x - 24 \)
56) \( f(x) = 2x^3 + 15x^2 + 24x \)
57) \( f(x) = \frac{4}{3}x^3 - 12x^2 + 10x + 50 \)
58) \( f(x) = x^4 - 24x^2 \)
59) \( f(x) = 10x^3 - 3x^5 \)
60) \( f(x) = \frac{1}{2}x^4 - 3x^3 + 12 \)
61) \( f(x) = \frac{1}{4}x^4 - x^3 + 15 \)
62) \( f(x) = \frac{5x}{x^2 + 36} \)
63) \( f(x) = (x + 3)^{2/3} - 8 \)
64) \( f(x) = x\sqrt{25 - x^2} \)

Determine where the given function is increasing and where it is decreasing.

65) \( s(x) = -x^2 - 22x - 40 \)
66) \( y = x^3 - 3x^2 + 6x - 8 \)
67) \( f(x) = x^3 - 12x + 4 \)
68) \( f(x) = 2x^3 + 12x^2 + 18x \)
69) \( f(x) = x^4 - 32x^2 - 6 \)
70) \( f(x) = (x + 2)^{2/3} - 5 \)
71) \( f(x) = (x - 3)^{1/3} + 7 \)
72) \( f(x) = \frac{7}{x^2 + 1} \)
73) \( f(x) = 80x^3 - 3x^5 \)

Determine where the given function is concave up and where it is concave down.
74) \( f(x) = x^2 - 18x + 88 \)
75) \( q(x) = 2x^3 + 2x + 9 \)
76) \( f(x) = x^3 + 3x^2 - x - 24 \)
77) \( G(x) = \frac{1}{4}x^4 - x^3 + 10 \)
78) \( f(x) = 2x^3 + 3x^2 - 12x \)
79) \( f(x) = x^4 - 24x^2 \)
80) \( f(x) = 10x^3 - 3x^5 \)
81) \( f(x) = x\sqrt{81 - x^2} \)

Solve the problem.
82) The annual revenue and cost functions for a manufacturer of grandfather clocks are approximately \( R(x) = 480x - 0.02x^2 \) and \( C(x) = 200x + 100,000 \), where \( x \) denotes the number of clocks made. What is the maximum annual profit?
83) The annual revenue and cost functions for a manufacturer of precision gauges are approximately \( R(x) = 500x - 0.02x^2 \) and \( C(x) = 120x + 100,000 \), where \( x \) denotes the number of gauges made. What is the maximum annual profit?
84) The percent of concentration of a certain drug in the bloodstream \( x \) hr after the drug is administered is given by \( K(x) = \frac{3x}{x^2 + 25} \). How long after the drug has been administered is the concentration a maximum? Round answer to the nearest tenth, if necessary.
A person coughs when a foreign object is in the windpipe. The velocity of the cough depends on the size of the object. Suppose a person has a windpipe with a 15-mm radius. If a foreign object has a radius \( r \), in mm, assume that the velocity \( V \), in mm/second, needed to remove the object by a cough is given by

\[
V(r) = k(15r^2 - r^3), \quad 0 \leq r \leq 15,
\]

where \( k \) is some positive constant. For what size object is the maximum velocity needed to remove the object? Round answer to the nearest tenth, if necessary.

Because of material shortages, it is increasingly expensive to produce 6.0L diesel engines. In fact, the profit in millions of dollars from producing \( x \) hundred thousand engines is approximated by \( P(x) = -x^3 + 29x^2 + 16x - 31 \), where \( 0 \leq x \leq 20 \). Find the inflection point of this function to determine the point of diminishing returns.

The function \( R(x) = 10,000 - x^3 + 27x^2 + 600x \), \( 0 \leq x \leq 20 \), represents revenue in thousands of dollars where \( x \) represents the amount spent on advertising in tens of thousands of dollars. Find the inflection point for the function to determine the point of diminishing returns.

Graph the equation. Include the coordinates of any local and absolute extreme points and inflection points.

Sketch the graph and show all local extrema and inflection points.

Graph the equation. Include the coordinates of any local and absolute extreme points and inflection points.

Find the absolute maximum and absolute minimum values of the function, if they exist, on the indicated interval. When no interval is specified, use the real line \((-\infty, \infty)\).

Find the absolute maximum and absolute minimum values of the function, if they exist, over the indicated interval.
97) \( f(x) = x^3 - 3x + 3; [-4, 1] \)

98) \( f(x) = x^4 - 32x^2 + 5; [-5, 5] \)

99) \( f(x) = 6 - x^{2/3}; [-1, 1] \)

100) \( f(x) = -\frac{2}{x^2}; [0.5, 3] \)

101) \( f(x) = -x^2 + 12x - 35; [5, 7] \)

**Solve the problem.**

102) \( P(x) = -x^3 + \frac{27}{2}x^2 - 60x + 100, x \geq 5 \) is an approximation to the total profit (in thousands of dollars) from the sale of \( x \) hundred thousand tires. Find the number of tires that must be sold to maximize profit.

103) \( S(x) = -x^3 + 6x^2 + 288x + 4000, 4 \leq x \leq 20 \) is an approximation to the number of salmon swimming upstream to spawn, where \( x \) represents the water temperature in degrees Celsius. Find the temperature that produces the maximum number of salmon. Round to the nearest tenth, if necessary.

104) The cost of a computer system increases with increased processor speeds. The cost \( C \) of a system as a function of processor speed is estimated as \( C = 5S^2 - 8S + 1100 \), where \( S \) is the processor speed in MHz. Find the processor speed for which cost is at a minimum.

105) Assume that the temperature \( T \) of a person during a certain illness is given by \( T(t) = -0.1t^2 + 1.4t + 98.6, 0 \leq t \leq 12 \) where \( T \) = the temperature (°F) at time \( t \), in days. Find the maximum value of the temperature and when it occurs. Round your answer to the nearest tenth, if necessary.

106) The total-revenue and total-cost functions for producing \( x \) clocks are \( R(x) = 520x - 0.02x^2 \) and \( C(x) = 160x + 100,000 \), where \( 0 \leq x \leq 25,000 \). What is the maximum annual profit?

107) For a simply supported beam with a load that increases uniformly from left to right, the bending moment \( M \) (in ft-lb) at a distance of \( x \) (in ft) from the left end is given by \( M = \frac{1}{6}(w)2x - wx^3 \). Determine the location of the maximum bending moment. In the formula, \( w \) is the rate of load increase \( \left[ \text{in lb/ft} \right] \) and \( l \) is the length (in ft) of the beam.

108) Find the maximum profit given the following revenue and cost functions:

\[ R(x) = 132x - x^2 \]
\[ C(x) = \frac{1}{3}x^3 - 9x^2 + 96x + 33 \]

where \( x \) is in thousands of units and \( R(x) \) and \( C(x) \) are in thousands of dollars.
109) An appliance company determines that in order to sell $x$ dishwashers, the price per
dishwasher must be
\[ p = 600 - 0.3x. \]

It also determines that the total cost of producing $x$ dishwashers is given by
\[ C(x) = 3000 + 0.9x^2. \]

What price must be charged per dishwasher in order to maximize profit?

110) A hotel has 300 units. All rooms are occupied when the hotel charges $80 per day for a
room. For every increase of $x$ dollars in the daily room rate, there are $x$ rooms vacant. Each
occupied room costs $36 per day to service and maintain. What should the hotel charge
per day in order to maximize daily profit?

111) An outdoor sports company sells 300 kayaks per year. It costs $12 to store one kayak for a
year. Each reorder costs $8, plus an additional $7 for each kayak ordered. How many
times per year should the store order kayaks in order to minimize inventory costs?

112) If the price charged for a bolt is $p$ cents, then $x$ thousand bolts will be sold in a certain
hardware store, where $p = 24 - \frac{x}{28}$. How many bolts must be sold to maximize revenue?

113) A baseball team is trying to determine what price to charge for tickets. At a price of $10
per ticket, it averages 40,000 people per game. For every increase of $1, it loses 5,000
people. Every person at the game spends an average of $5 on concessions. What price per
ticket should be charged in order to maximize revenue?

Find the elasticity of the demand function as a function of $p$.

114) $x = D(p) = \frac{400}{p}$

115) $x = D(p) = \sqrt{400 - p}$

Find the elasticity of the demand function at the given price and state whether the demand is elastic, inelastic, or
whether it has unit elasticity.

116) $q = D(p) = 300 - 4p; \$47$

117) $q = D(p) = \sqrt{690 - p}; \$540$

Solve the problem.

118) A beverage company works out a demand function for its sale of soda and finds it to be
$x = D(p) = 4100 - 26p,$

where $x =$ the quantity of sodas sold when the price per can, in cents, is $p$. At what price is
the revenue a maximum?
119) A CD store determines the following demand function for a particular CD

\[ x = D(p) = \sqrt{220 - p^2}, \]

where \( x \) = the number of CDs sold per day when the price per CD is \( p \) dollars. At what price is the revenue a maximum? Round your answer to the nearest dollar.

120) The stadium vending company finds that sales of hot dogs average 36,000 hot dogs per game when the hot dogs sell for $2.50 each. For each 50 cent increase in the price, the sales per game drop by 5000 hot dogs. What price per hot dog should the vending company charge to realize the maximum revenue?

121) A truck burns fuel at the rate (gallons per mile) of

\[ G(x) = \frac{1}{26} \left( \frac{64}{x} + \frac{x}{36} \right) \]

while traveling at \( x \) mph. If fuel costs $1.33 per gallon, find the speed that minimizes fuel cost for a 200-mile trip.
1) $0.48
2) $6.60
3) $1.04
4) 30.00 dollars
5) -60 thousand dollars
6) $4558
7) $63,801
8) 82
9) D'(p) = -8p + 8
10) \( \frac{2}{\pi} \) in.
11) 1200 mm³
12) \(-\frac{9x}{8y}\)
13) \(\frac{x^2}{6y}\)
14) \(\frac{1 - y}{2 + x}\)
15) \(\frac{2x - y}{x - 2y}\)
16) \(\frac{x}{y}\)
17) \(\frac{9y}{-9x + 7}\)
18) \(\frac{21x^2 - 2xy^3}{3x^2y^2}\)
19) \(-\frac{x^2 + 2xy}{x^2 + y^2}\)
20) \(\frac{2xy^2 - y - 1}{-2x^2y + x + 1}\)
21) \(-(x/y)^{1/3}\)
22) \(\frac{x(x - y)^2 + y}{x - y(x - y)^2}\)
23) \(\frac{y}{2(x + 1)}\)
24) $500.5
25) $26,458/month
26) 32.04 ft/s²
27) 4%/min
28) $133 per day
29) \(\frac{9}{160}\) ohms/s
30) Relative minimum at (5, 10)
31) Relative maximum at \(\left( -\frac{1}{12}, -\frac{71}{12} \right)\)
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32) Relative maximum at (0, 1); relative minimum at (2, -3)
33) No relative extrema exist
34) Relative maximum at (-2, 11); relative minimum at (2, -21)
35) Relative maximum at \(-1, \frac{16}{3}\); relative minimum at \(3, -16\)
36) Relative minimum at (0, 32)
37) Relative maximum at \(-1, 16\); relative minimum at (2, -21)
38) No relative extrema exist
39) Relative maximum at (0, -2)
40) Relative minimum at (0, -3)
41) Relative minimum at \(-1, -3\); relative maximum at \(1, 3\)
42) Relative maximum at \(0, \frac{1}{2}\); relative minimum at \(-2, -\frac{1}{2}\)
43) Relative minimum at (0, -1)
44) No relative extrema exist
45) Relative minimum at (-5, -2)
46) Relative minimum at (-6, 6)
47) (0, 12), (225, 50,637), (450, 12)
48) (0, 98.6), (6.5, 102.8), (13, 98.6)
49) (175, 67.95)
50) (0, 6)
51) (0, 1)
52) (0, 1)
53) (0, 0)
54) (1, 15)
55) (4, 108)
56) \(-\frac{5}{2}, \frac{5}{2}\)
57) (3, 8)
58) (-2, -80), (2, -80)
59) (0, 0), (-1, -7), (1, 7)
60) (0, 12), (3, -28.5)
61) (0, 15), (2, 11)
62) (0, 0), \left(\sqrt{108}, \frac{5}{144}\sqrt{108}\right), \left(-\sqrt{108}, -\frac{5}{144}\sqrt{108}\right)
63) No points of inflection exist
64) (0, 0)
65) Increasing on \((-\infty, -11]\), decreasing on \([-11, \infty)\)
66) Increasing on \((-\infty, \infty)\)
67) Increasing on \((-\infty, -2]\) and \([2, \infty)\), decreasing on \([-2, 2]\)
68) Increasing on \((-\infty, -3]\) and \([-1, \infty)\), decreasing on \([-3, -1]\)
69) Decreasing on \((-\infty, -4]\) and \([0, 4]\), increasing on \([-4, 0]\) and \([4, \infty)\)
70) Decreasing on \((-\infty, -2]\), increasing on \([-2, \infty)\)
71) Increasing on \((-\infty, \infty)\)
72) Increasing on \((-\infty, 0]\), decreasing on \([0, \infty)\)
73) Decreasing on \((-\infty, -4]\) and \([4, \infty)\), increasing on \([-4, 4]\)
74) Concave up for all x
75) Concave up on \((0, \infty)\), concave down on \((-\infty, 0)\)
76) Concave up on \((-1, \infty)\), concave down on \((-\infty, -1)\)
77) Concave up on \((\infty, 0)\) and \((2, \infty)\), concave down on \((0, 2)\)

78) Concave down on \([-\infty, -\frac{1}{2}]\) concave up on \([\frac{1}{2}, \infty)\)

79) Concave up on \((-\infty, -2)\) and \((2, \infty)\), concave down on \((-2, 2)\)

80) Concave up on \((-\infty, -1)\) and \((0, 1)\), concave down on \((-1, 0)\) and \((1, \infty)\)

81) Concave up on \((-\infty, -9)\), concave down on \((9, \infty)\)

82) $880,000

83) $1,705,000

84) 5 hr

85) 10

86) \((9.67, 1930.26)\)

87) \((9, 16,858)\)

88) local minimum: \((1, -7)\)

local maximum: \((-2, 20)\)

inflection point: \((-\frac{1}{2}, \frac{\sqrt{3}}{2})\)

89) Absolute maxima: \((-1, -9), (1, -9)\)

Local minimum: \((0, -10)\)

Inflection points: \((-\frac{\sqrt{3}}{3}, 1), (\frac{\sqrt{3}}{3}, 1)\)
Answer Key
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90) Absolute maxima: (-1, -3), (1, -3)
   Local minimum: (0, -4)
   Inflection points: \([-\frac{1}{\sqrt{3}}, \frac{1}{\sqrt{3}}]\)

91) local minimum: (3, 0)
   local maximum: (1, 8)
   inflection point: (2, 4)

92) Absolute maximum: \(\frac{256}{675}\), absolute minimum: -936

93) Absolute maximum: 32, absolute minimum: -\(\frac{27}{16}\)

94) Absolute maximum: -8, absolute minimum: -17

95) Absolute maximum: 48, absolute minimum: -48

96) Absolute maximum: 23, absolute minimum: -\(\frac{4431}{2}\)

97) Absolute maximum: 5, absolute minimum: -49

98) Absolute maximum: 5, absolute minimum: -251

99) Absolute maximum: 6, absolute minimum: 5

100) Absolute maximum: -\(\frac{2}{9}\), absolute minimum: -8

101) Absolute maximum: 1; absolute minimum: 0
102) 500,000
103) 12°C
104) 0.8 MHz
105) 103.5°F at 7 days
106) $1,520,000
107) \( x = \frac{1\sqrt{3}}{3} \)
108) 1263 thousand dollars
109) $525
110) $208
111) 15
112) 336 thousand bolts
113) $6.50
114) \( E(p) = 1 \)
115) \( E(p) = \frac{p}{800 - 2p} \)
116) $\frac{47}{28}$; elastic
117) $\frac{9}{5}$; elastic
118) 79 cents
119) $10$
120) $3.05$
121) 48 mph