Exam 4 will be on 07/23/12 and covers the following sections: 5.4, 5.5, 5.6, 6.2, 6.4.

SHORT ANSWER. Write the word or phrase that best completes each statement or answers the question.

Find the area of the shaded region.

1) \( y = x^2 - 4x + 3 \)   
   \[ y = x - 1 \]   
   \[ y = x^2 \]   
   \[ y = -1 \]

2) \( y = 2x^2 + x - 6 \)   
   \( y = x^2 - 4 \)

3) \( y = \sqrt{2x} \)   
   \( y = x - 4 \)
4) \[ y = x^4 - 32 \]

Find the area enclosed by the given curves.
5) \[ y = x^3, \ y = 4x \]
6) \[ y = 2x - x^2, \ y = 2x - 4 \]
7) \[ y = x, \ y = x^2 \]
8) \[ y = \frac{1}{2} x^2, \ y = -x^2 + 6 \]

MULTIPLE CHOICE. Choose the one alternative that best completes the statement or answers the question.

9) Find the area bounded by the curves \( y = x^2 - 2x \) and \( y = 3 \).
   A) 10 \quad B) \frac{32}{3} = 10\frac{2}{3} \quad C) 9 \quad D) \frac{20}{3} = 6\frac{2}{3}

SHORT ANSWER. Write the word or phrase that best completes each statement or answers the question.

For the problems below, find each area bounded by the curves.
10) \[ y = 2x^2 - 5x - 8 \]
   \[ y = x + 12 \]

11) \[ y = x^3 \]
   \[ y = 2x \]

12) What is the consumers’ surplus for the demand curve \( p = 5 - \frac{x}{20} \) at the sales level \( x = 60 \)?

13) Find the producers’ surplus for the supply curve \( p = 0.02x + 7 \) at \( x = 100 \).

14) Find the producers’ surplus for the supply curve \( p = 4 + \frac{1}{2}\sqrt{x} \) at \( x = 144 \).
Find the consumer's surplus for the following demand function at the given point.

15) \( D(x) = (x - 3)^2; x = \frac{3}{2} \)

16) Find the consumers' surplus at a price level of \( p = $7 \) for the price-demand equation \( p = D(x) = 25 - 0.4x \).

Find the producer's surplus for the following supply function at the given point.

17) \( S(x) = x^2 + 5; x = 1 \)

Solve the problem.

18) Find the consumers' surplus and producers' surplus for \( p = D(x) = 71 - \frac{1}{10}x \) and \( p = S(x) = 35 + \frac{1}{20}x \).

19) Find the producers' surplus at a price level of \( p = $30 \) for the price-supply equation \( p = S(x) = 14 + 0.0004x^2 \).

Solve the problem.

20) Find the equilibrium price if the price-demand equation is \( p = D(x) = 23 - \frac{1}{20}x \), and the price-supply equation is \( p = S(x) = 8 + \frac{1}{8,000}x^2 \).

21) Find the equilibrium quantity if the price-demand equation is \( p = D(x) = 23 - \frac{1}{20}x \), and the price-supply equation is \( p = S(x) = 8 + \frac{1}{8,000}x^2 \).

22) Find the equilibrium price and quantity, producers' surplus for \( p = D(x) = 71 - \frac{1}{10}x \) and \( p = S(x) = 35 + \frac{1}{20}x \).

Solve the differential equation.

23) \( \frac{dy}{dx} = e^{6x} - 6y \)

24) \( \frac{dy}{dx} = \frac{5y^2}{x} \)

25) \( \frac{dy}{dx} = 12\sqrt{xy} \)
26) \(x^2 \frac{dy}{dx} = 2y\)  
27) \(\frac{dy}{dx} = 4x^3e^{-y}\)  
28) \(\frac{dy}{dx} = 3x^2\sqrt{y - 1}\)

Find a general solution for the differential equation.

29) \(y' = \frac{2y^2}{x}\)
30) \(y' = 18\sqrt{xy}\)
31) \(x^2y' = 2y\)
32) \(y' = 5x^4e^{-y}\)
33) \(\frac{dy}{dx} = 7x^6\sqrt{y - 1}\)

Solve the initial-value problem.

34) \(2y' - 4xy = 8x; \ y(0) = 24\)
35) \(\frac{dy}{dx} - xy - x = 0; \ y(1) = 10\)
36) \(y' = e^x - y; \ y(0) = 3\)
37) \(y' = \frac{3 - x^3}{3y + 8}; \ y(0) = 2\)

Solve the problem.

38) If the marginal price \(\frac{dp}{dx}\) at \(x\) units of demand per week is proportional to the price \(p\), and if at $80 there is no weekly demand \([p(0) = 80]\), and if at $50.18 there is a weekly demand of 8 units \([p(8) = 50.18]\), find the price-demand equation.

39) Find the amount \(A\) in an account (to the nearest dollar) after 5 years if \(\frac{dA}{dt} = rA\), \(A(0) = 800\), and \(A(10) = 1800\).
40) A single injection of a drug is administered to a patient. The amount $Q$ in the body then decreases at a rate proportional to the amount present, and for this particular drug the rate is 3% per hour. Thus, $\frac{dQ}{dt} = -0.03Q$ with $Q(0) = Q_0$, where $t$ is time in hours. If the initial injection is 4 milliliters [$Q(0) = 4$], about how many hours after the drug is given will there be 2 milliliters of the drug remaining in the body? (Round answer to the nearest tenth of an hour.)

41) At the beginning of an advertising campaign for a new product in a city of 500,000 people, no one is aware of the product. After 10 days, 100,000 people are aware of the product. If $N = N(t)$ is the number of people (in thousands) who are aware of the product $t$ days after the beginning of the advertising campaign, solve the following differential equation for $N(t)$:

$$\frac{dN}{dt} = k(500 - N); \quad N(0) = 0; \quad N(10) = 100.$$ 

42) A computer manufacturer finds that the marginal supply for its new laptop computer satisfies the function

$$S'(p) = \frac{110p}{(27 - p)^2},$$

where $S$ is the quantity purchased when the price is $p$ hundred dollars. Find the supply function $S(p)$ given that the company will sell 730 computers when the price is 18 hundred dollars.

43) The rate of change of the population of a town is given by

$$P'(t) = \frac{2.3t}{t - 6}, \quad t > 10,$$

where $P$ is the population in thousands $t$ years after 1970. Find the function $P(t)$ given that the population in 1999 is 176 thousand.

44) A car manufacturer finds that the marginal supply for its new station wagon satisfies the function

$$S'(p) = \frac{-8600}{p(p - 12)},$$

where $S$ is the quantity purchased in one town when the price is $p$ thousand dollars. Find the supply function $S(p)$ given that the company will sell 640 cars when the price is 25 thousand dollars.
45) The rate of change of the population of a town is given by

\[ P'(t) = \frac{1.5t}{t-5}, \quad t > 10, \]

where \( P \) is the population in thousands \( t \) years after 1970. Find the function \( P(t) \) given that the population in 1999 is 135 thousand.

46) A car manufacturer finds that the marginal supply for its new station wagon satisfies the function

\[ S'(p) = \frac{-7200}{p(p-12)}, \]

where \( S \) is the quantity purchased in one town when the price is \( p \) thousand dollars. Find the supply function \( S(p) \) given that the company will sell 601 cars when the price is 24 thousand dollars.
Answer Key
Testname: MAC_2233_SUMMER_C_EXAM_4_REVIEW

1) $\frac{19}{2}$
2) $\frac{19}{3}$
3) $\frac{64}{3}$
4) $\frac{512}{5}$
5) 8
6) $\frac{32}{3}$
7) $\frac{1}{6}$
8) 16
9) B
10) $114\frac{1}{3}$
11) 2
12) 90
13) $100$
14) $288$
15) $4.50$
16) $405$
17) $0.67$
18) CS = $2880
    PS = $1440
19) $2133$
20) $13.00$
21) 200
22) $p = 47$
    $q = 240$
23) $y = \frac{1}{6} \ln (e^{6x} + C)$
24) $y = \frac{-1}{5 \ln x + C}$
25) $y = (4x^{3/2} + C)^2$
26) $y = C e^{-2/x}$
27) $y = \ln (x^4 + C)$
28) $y = \left(\frac{1}{2} x^3 + C\right)^2 + 1$
29) $y = \frac{-1}{2 \ln x + C}$
30) $y = (6x^{3/2} + C)^2$
31) $y = C e^{-2/x}$
32) $y = \ln (x^5 + C)$
33) $y = \left(\frac{1}{2} x^7 + C\right)^2 + 1$
34) \( y = -2 + 26e^{x^2} \)

35) \( y = -1 + 11e^{(x^2 - 1)/2} \)

36) \( y = \ln(e^x + e^3 - 1) \)

37) \( \frac{3}{2}y^2 + 8y = 3x - \frac{1}{4}x^4 + 22 \)

38) \( p(x) = 80e^{-0.058x} \)

39) \$1200 \)

40) 23.1 hours

41) \( N(t) = 500(1 - e^{-0.022t}) \)

42) \( S(p) = \frac{2970}{27 - p} - 110 \ln |27 - p| + 158 \)

43) \( P(t) = 2.3t + 13.8 \ln (t - 6) + 66 \)

44) \( S(p) = \frac{2150}{3} \ln \left| \frac{p}{p - 12} \right| + 171 \)

45) \( P(t) = 1.5t + 7.5 \ln (t - 5) + 68 \)

46) \( S(p) = 600 \ln \left| \frac{p}{p - 12} \right| + 185 \)