Factoring Polynomials

Factoring polynomials of the form $ax^2 + bx + c$, $(a \neq 1)$ is usually done by trial and error.

**Example:** Factor $3x^2 + 17x + 10$.

Let $F O I L = ax^2 + adx + bc + bd$.

Notice $ac = 3$, $ad + bc = 17$ and $bd = 10$.

Since $a$ and $c$ are the factor pairs of 3 and $b$ and $d$ are the factor pairs of 10.

We have to look at all possible combinations of binomial factors.

<table>
<thead>
<tr>
<th>Factor pairs of $3$</th>
<th>Factor pairs of $5$</th>
</tr>
</thead>
<tbody>
<tr>
<td>3 and 1</td>
<td>1 and 10</td>
</tr>
<tr>
<td>3 and 5</td>
<td>2 and 5</td>
</tr>
</tbody>
</table>

$= (3x + 10)(x + 1) = (3x^2 + 3x + 10x + 10) = (3x^2 + 13x + 10)$

$= (3x + 1)(x + 10) = (3x^2 + 30x + 1x + 10) = (3x^2 + 31x + 10)$

$= (3x + 5)(x + 2) = (3x^2 + 15x + 2x + 10) = (3x^2 + 17x + 10)$

Example: Factor $8x^2 - 6x - 9$.

Let $F O I L = ax^2 + adx + bc + bd$.

Notice $ac = 8$, $ad + bc = -6$ and $bd = -9$.

Since $a$ and $c$ are the factor pairs of 8 and $b$ and $d$ are the factor pairs of -9.

We have to look at all possible combinations of binomial factors.

<table>
<thead>
<tr>
<th>Factor pairs of $4$</th>
<th>Factor pairs of $3$</th>
</tr>
</thead>
<tbody>
<tr>
<td>2 and 4</td>
<td>1 and 9</td>
</tr>
<tr>
<td>2 and 8</td>
<td>3 and 3</td>
</tr>
</tbody>
</table>

$= (4x + 9)(2x - 1) = (8x^2 - 9x + 8x - 9) = (8x^2 - 1x - 9)$

$= (4x - 9)(2x + 1) = (8x^2 + 1x - 72x - 9) = (8x^2 - 71x - 9)$

$= (4x - 3)(2x + 3) = (8x^2 - 12x - 6x - 9) = (8x^2 - 21x - 9)$

$= (2x + 3)(4x - 3) = (8x^2 - 6x + 12x - 9) = (8x^2 + 6x - 9)$

$= (2x + 3)(2x + 3) = (8x^2 + 6x - 12x - 9) = (8x^2 - 6x - 9)$
Example: Factor $4x^2 - 12x + 5$. 

Notice $ac = 4$, $ad + bc = -12$ and $bd = 5$. 

$$= \left( \frac{ax}{x} + \frac{b}{d} \right) \left( \frac{cx}{x} + \frac{d}{c} \right)$$

Factor parts of $4$

1 and 4

2 and 2

Factor parts of $5$

1 and 5

$$= (4x - 1)(4x - 5) = (4x^2 - 5x - 4x + 5) = (4x^2 - 9x + 5)$$

$$= (4x - 1)(4x - 1) = (4x^2 - 4x - 10x + 5) = (4x^2 - 21x + 5)$$

$$= (2x - 1)(2x - 3) = (4x^2 - 2x - 10x + 5) = (4x^2 - 12x + 5)$$

$$= (2x - 3)(2x - 1) = (4x^2 - 2x - 6x + 3) = (4x^2 - 12x + 3)$$

Factoring polynomials of the form $ax^2 + bx + c$, $(a \neq 1)$ can also be done by grouping.

Example: Factor $4x^2 - 12x + 5$.

1. To begin multiply, $a \cdot c = 20$. Then we have to find factors of the product that sum to $b$.

<table>
<thead>
<tr>
<th>fac1</th>
<th>fac2</th>
<th>fac1+fac2</th>
</tr>
</thead>
<tbody>
<tr>
<td>-1</td>
<td>-20</td>
<td>-21</td>
</tr>
<tr>
<td>-2</td>
<td>-10</td>
<td>-12</td>
</tr>
<tr>
<td>-4</td>
<td>-5</td>
<td>-9</td>
</tr>
</tbody>
</table>

2. Replace the middle term with the sum of the appropriate factors

$$= (4x^2 - 2x - 10x + 5)$$ Replace the middle term.

$$= (4x^2 - 2x) - (10x - 5)$$ Group terms.

$$= 2(2x - 1) - 5(2x - 1)$$ Factor each pair of terms.

$$= (2x - 1)(2x - 5)$$ Factor out the common binomial.

Check: $(2x - 1)(2x - 5) = 4x^2 - 10x - 2x + 5 = 4x^2 - 12x + 5$.

Factoring polynomials of the form $ax^2 + bx + c$, $(a \neq 1)$ can also be done by grouping.

Example: Factor $2y^2 - y - 6$.

1. To begin multiply, $a \cdot c = -12$. Then we have to find factors of the product that sum to $b$.

<table>
<thead>
<tr>
<th>fac1</th>
<th>fac2</th>
<th>fac1+fac2</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>-12</td>
<td>-11</td>
</tr>
<tr>
<td>2</td>
<td>-6</td>
<td>-4</td>
</tr>
<tr>
<td>3</td>
<td>-4</td>
<td>-1</td>
</tr>
</tbody>
</table>

2. Replace the middle term with the sum of the appropriate factors

$$= (2y^2 + 3y - 4y - 6)$$ Replace the middle term.

$$= (2y^2 + 3y) - (4y + 6)$$ Group terms.

$$= y(2y + 3) - 2(2y + 3)$$ Factor each pair of terms.

$$= (2y + 3)(y - 2)$$ Factor out the common binomial.

Check: $(2y + 3)(y - 2) = 2y^2 - 4y + 3y - 6 = 2y^2 - y - 6$.
**Example:** Factor \(2x^2 + 5x + 3\) by grouping.

1. To begin multiply, \(a \cdot c = 6\), then we have to find factors of the product that sum to \(b\).
2. Replace the middle term with the sum of the appropriate factors
3. Factor by Grouping

\[
\begin{array}{c|c|c|c}
\text{Fact 1} & \text{Fact 2} & \text{Fact 1+Fact 2} \\
\hline
1 & 6 & 15 \\
\hline
2 & 3 & 5 \\
\end{array}
\]

\[
2x^2 + 5x + 3 = (2x^2 + 3x + 2x + 3) \quad \text{Replace the middle term.}
= (2x + 3)(x + 1) \quad \text{Group terms.}
= x(2x + 3) + 1(2x + 3) \quad \text{Factor each pair of terms.}
= (2x + 3)(x + 1) \quad \text{Factor out the common binomial.}
\]

Check: \((2x + 3)(x + 1) = 2x^2 + 2x + 3x + 3 = 2x^2 + 5x + 3.\)

---

**Example:** Factor \(2x^2 + 5x + 3\) by grouping.

1. To begin multiply, \(a \cdot c = 6\), then we have to find factors of the product that sum to \(b\).
2. Replace the middle term with the sum of the appropriate factors
3. Factor by Grouping

\[
\begin{array}{c|c|c|c}
\text{Fact 1} & \text{Fact 2} & \text{Fact 1+Fact 2} \\
\hline
1 & 6 & 15 \\
\hline
2 & 3 & 5 \\
\end{array}
\]

\[
2x^2 + 5x + 3 = (2x^2 + 3x + 2x + 3) \quad \text{Replace the middle term.}
= (2x + 3)(x + 1) \quad \text{Group terms.}
= x(2x + 3) + 1(2x + 3) \quad \text{Factor each pair of terms.}
= (2x + 3)(x + 1) \quad \text{Factor out the common binomial.}
\]

Check: \((2x + 3)(x + 1) = 2x^2 + 2x + 3x + 3 = 2x^2 + 5x + 3.\)

---

**Trinomials which are quadratic in form are factored like quadratic trinomials.**

**Example:** Factor \(3x^4 + 28x^2 + 9\).

\[
3x^4 + 28x^2 + 9 = 3a^2 + 28a + 9
= (3a + 1)(a + 9) \quad \text{Factor.}
= (3a + 1)(a + 9)
\]

**Example:** Factor \(x^2 + 3x + 5\).

Let \(x^2 + 3x + 5 = (x + a)(x + b) = x^2 + (a + b)x + ab\).

Then \(a + b = 3\) and \(ab = 5\). This is impossible.

The trinomial \(x^2 + 3x + 5\) cannot be factored.