Factoring Polynomials

Special Rules

A difference of squares can be factored using the formula

\[ a^2 - b^2 = (a + b)(a - b). \]

Example: Factor \( x^2 - 9y^2 \).

\[ x^2 - 9y^2 = (x)^2 - (3y)^2 \quad \text{Write terms as perfect squares.} \]
\[ = (x + 3y)(x - 3y) \quad \text{Use the formula.} \]

The same method can be used to factor any expression which can be written as a difference of squares.

Example: Factor \( 4(x + 1)^2 - 25y^4 \).

\[ 4(x + 1)^2 - 25y^4 = (2(x + 1))^2 - (5y^2)^2 \]
\[ = [(2(x + 1)) + (5y^2)][(2(x + 1)) - (5y^2)] \]
\[ = (2x + 2 + 5y^2)(2x + 2 - 5y^2) \]

Note: The sum of squares \((a^2 + b^2)\) cannot be factored.

Perfect Square Trinomials can be factored using the formulas

\[ a^2 + 2ab + b^2 = (a + b)^2. \]
\[ a^2 - 2ab + b^2 = (a - b)^2. \]

Example: Factor \( 9x^2 + 30x + 25 \).

\[ 9x^2 + 30x + 25 = (3x)^2 + 2(3x)(5) + (5)^2 \quad \text{Write terms as perfect squares.} \]
\[ = (3x + 5)^2 \quad \text{Use the formula.} \]

Example: Factor \( 36m^2 - 60mn + 25n^2 \).

\[ = (6m)^2 - 2(6m)(5n) + (5n)^2 \]
\[ = (6m - 5n)^2 \]
A difference of cubes can be factored using the formula
\[ a^3 - b^3 = (a - b)(a^2 + ab + b^2). \]

**Example:** Factor \(8x^3 - 27\).
\[ 8x^3 - 27 = (2x)^3 - (3)^3 \]
\[ = (2x - 3)(4x^2 + 6x + 9) \]
Write terms as perfect cubes.
Use the formula.

A similar method can be used to factor any expression which is
the sum of cubes.
\[ a^3 + b^3 = (a + b)(a^2 - ab + b^2). \]

**Example:** Factor \(n^3 + 64\).
\[ n^3 + 64 = (n)^3 - (4)^3 \]
Write terms as perfect cubes.
\[ = (n - 4)(n^2 + 4n + 16) \]
Use the formula.