Introduction

What are Polynomials?
A polynomial in $x$ consists of a finite number of terms of the form $ax^n$ where $a$ can be any Real number but $n$ must be a whole number. (Recall a term is any algebraic expression separated from another algebraic expression by “+” or “-” signs. Whole numbers are $\{0,1,2,3,4,...\}$)
The following are examples of polynomials:

\[2x^4\] A one term polynomial is called a **monomial**.

\[-5x^6 + 7.9x^2\] A two term polynomial is called a **binomial**.

\[2x^2 + 3x - 1\] A three term polynomial is called a **trinomial**.

The following are **not** polynomials:

\[3x^{-2} - 4x + 2\] is not a polynomial because the exponents on the variables are not positive whole numbers.

\[19y^{1/2} + 5\] is not a polynomial because the exponent on the variable is not a whole number.

The following are examples of polynomials in more than one variable:

\[2x^4y^2\] is a monomial in \(x\) and \(y\)

\[-5x^6yz + 7.9x^2yz^2\] is a binomial in \(x\), \(y\) & \(z\)

\[2x^2w + 3xw^2 - w^3\] is a trinomial in \(x\) and \(w\)
How many terms does the following polynomial have?

$7x^4 - 3x^2 + 4x^3 - 9x + 5$

The polynomial $7x^4 - 3x^2 + 4x^3 - 9x + 5$ has 5 terms.

Descending Powers

Writing a polynomial in descending powers means to begin with the term having the largest exponent on the variable and then proceeding to the lowest.

For example: $-3x^2 + 4x^3 - 9x + x^4 + 5$ would be written $x^4 + 4x^3 - 3x^2 - 9x + 5$
Degree of a Term
The degree of a term is the sum of the exponents on all variables.

For example: the degree of \(5x^2y^3z\) is \((2 + 3 + 1)\) or 6

For the polynomial \(x^4 + 4x^3 - 3x^2 - 9x + 5\) the degree of each term from left to right is 4, 3, 2, 1, and 0.

The constant 5 is equal to \(5x^0\), thus it has degree 0.

Degree of a Polynomial
The degree of a polynomial is the largest degree of any one term. Thus in the preceding polynomial, \(x^4 + 4x^3 - 3x^2 - 9x + 5\), the degree would be 4.

What is the degree of
\(x^7 + 4x^8 - 3x^9 - 9x^3 + 5\)?
The degree of \( x^7 + 4x^8 - 3x^9 - 9x^3 + 5 \) is 9
the highest degree term.

What are Like Terms?
Like terms are terms with the same variables
raised to the same powers.
For example:
\( 5x^2y^3 \) is like \(-4y^3 x^2\) but is not like \(5y^3x\)
x is like .35x but is not like \(x^2\)

Which of the following pairs are pairs of like terms?

(A) 3xy and 2yz   (B) -2xyz and 5xyz
(C) 3x^2 and 4 x^3
The answer is (B) \(-2xyz\) is like \(5xyz\) because both have the same variables raised to the same powers or exponents.