Division of Polynomials

Dividing by a Monomial
If the divisor only has one term, split the polynomial up into a fraction for each term.

\[
\frac{18x^4 - 24x^3 + 6x^2 - 12x}{6x} = \frac{18x^4}{6x} - \frac{24x^3}{6x} + \frac{6x^2}{6x} - \frac{12x}{6x}
\]

Now reduce each fraction.

\[
\frac{3x^3}{1} - \frac{4x^2}{1} + \frac{x}{1} - \frac{2}{1} = 3x^3 - 4x^2 + x - 2
\]

Another Example
If the divisor only has one term, split the polynomial up into a fraction for each term.

\[
\frac{6x^5y^3 - 2x^3y^2 + 15x^2y - 12x + 3}{6x^3y} = \frac{6x^5y}{6x^3y} - \frac{2x^3y^2}{6x^3y} + \frac{15x^2y}{6x^3y} - \frac{12x}{6x^3y} + \frac{3}{6x^3y}
\]

Now reduce each fraction.

\[
\frac{x^2y^2}{3y} - \frac{1x}{6x^2y} + \frac{5}{3y} - \frac{2}{2x^2y} + \frac{1}{xy}
\]

\[
= x^2y^2 - \frac{x}{3y} + \frac{5}{2xy} - \frac{2}{2x^2y} + \frac{1}{xy}
\]
Another Example

If the divisor only has one term, split the polynomial up into a fraction for each term.

\[
\frac{18z^4 - 15z^3 + 6z^2 - 3z}{3z} = \frac{18z^4}{3z} - \frac{15z^3}{3z} + \frac{6z^2}{3z} - \frac{3z}{3z}
\]

Now reduce each fraction.

\[
\frac{6z^3}{3z} - \frac{5z^3}{3z} + \frac{2z}{3z} - \frac{1}{3z} = \frac{6z^3 - 5z^3 + 2z - 1}{3z}
\]

Dividing Polynomials Using Long Division

Long division of polynomials is similar to long division of whole numbers.

When you divide two polynomials you can check the answer using the following:

\[\text{dividend} = (\text{quotient} \times \text{divisor}) + \text{remainder}\]

The result is written in the form:

\[\frac{\text{dividend}}{\text{divisor}} = \frac{\text{quotient}}{\text{divisor}} + \frac{\text{remainder}}{\text{divisor}}\]

Example: Divide 10236 by 11 and check the answer.

\[
\begin{array}{c|ccccc}
11 & 10236 \\
-11 & \underline{100} & \underline{36} \\
\hline
& 33 & 6 \\
& 33 & \underline{6} \\
\hline
& & 6 \\
\end{array}
\]

Answer: 930 + 6/11

Check: \(\frac{930 \times 11 + 6}{11}\) correct
Example 1: Divide \((X^2 + 5X + 12)\) by \((X + 3)\)

Set up the long division problem.

Divide the first term \(X^2\) by the first term in the divisor, \(X\). Write the result above \(5X\).

Multiply \(X\) by the divisor \(X + 3\) and write the answer below the dividend matching like terms as you go.

Subtract the bottom line by changing the signs of the bottom line you just wrote. When finished bring down the next term, which is \(12\).

We are not finished yet so continue onto the next slide!

\[
\begin{array}{l}
\text{Divide (12 + X^2) by (X + 3)} \\
\text{In a long division problem you must follow two set-up rules.} \\
\text{1) The dividend must be arranged in descending powers. Thus} \\
\text{12 + X^2 must be written as X^2 + 12.} \\
\text{2) If there are any missing exponents in your dividend, you make} \\
\text{space for them by adding a zero term.} \\
\end{array}
\]
Example 2: Divide \((X^2 - 5)\) by \((X - 2)\)

Set up the long division problem.

\[
\begin{array}{c|cc}
& X^2 & 0X - 5 \\
X - 2 & \hline
& X & 0X - 5 \\
& X - 2 & \hline
& X & 0X - 5 \\
& -X & 2X - 5 \\
& X - 2 & \hline
& X^2 & 0X - 5 \\
& -X^2 + 2X & X - 5 \\
& 2X & 4 \\
& +2X & 5 \\
& & \hline
& & 0X - 1 \\
We are not finished yet so continue onto the next page!
\]

Divide the first term \(X^2\) by the first term in the divisor, \(X\). Write the result above \(0X\).

Multiply \(X\) by the divisor \(X - 2\) and write the answer below the dividend matching like terms as you go.

Subtract the bottom line by changing the signs of the bottom line you just wrote. When finished bring down the next term, which is \(-5\)

We are not finished yet so continue onto the next page!

Example 3: Divide \(\frac{8X^3 - 1}{2X + 1}\)

\[
\begin{array}{c|cc}
& 8X^3 & 0X^2 + 0X - 1 \\
2X + 1 & \hline
& 4X^2 & 0X^2 + 0X - 1 \\
& 4X^2 & \hline
& 8X^3 & 0X^2 + 0X - 1 \\
& 8X^3 + 4X^2 & 0X - 1 \\
& 8X^3 + 4X^2 & \hline
& & 0X - 1 \\
We are not finished yet so continue onto the next page!
\]

Divide the first term \(2X\) by the first term in the divisor, \(X\). The answer is 2. Write +2 above the -5.

Multiply \(2(X - 2) = 2X - 4\)

Write answer below \(2X - 5\).

Subtract by changing signs. The remainder is -1.

We are not finished yet so continue onto the next page!
Step 1: Divide \(-4X^3\) by \(2X\).

Step 2: Multiply \(-2X\) by divisor \(2x + 1\).

Step 3: Subtract signs and bring down left over terms.

Repeat Steps 1 - 3 again.

Example: Divide \(x^2 + 3x - 2\) by \(x + 1\) and check the answer.

Answer: \(x + 2 + \frac{-4}{x + 1}\)

Check: \((x + 2)(x + 1) + (-4) = x^2 + 3x - 2\) correct

quotient divisor remainder dividend
**Example:** Divide \(4x + 2x^3 - 1\) by \(2x - 2\) and check the answer.

\[
x^2 + x + 3
\]

\[
2x - 2 \left( \frac{2x^3 + 0x^2 + 4x - 1}{2x^3 - 2x^2} \right)
\]

\[
\frac{2x^2 + 4x}{2x^2 - 2x}
\]

\[
\frac{6x - 1}{6x - 6}
\]

\[\frac{5}{2x - 2}\]

Answer: \(x^2 + x + 3 + \frac{5}{2x - 2}\)

Check: \((x^2 + x + 3)(2x - 2) + 5 = 4x + 2x^3 - 1\)

---

**Example:** Divide \(x^2 - 5x + 6\) by \(x - 2\).

\[
x^2 - 3
\]

\[
x - 2 \left( \frac{x^2 - 5x + 6}{x^2 - 2x} \right)
\]

\[
-3x + 6
\]

\[
-3x + 6
\]

Answer: \(x - 3\) with no remainder.

Check: \((x - 2)(x - 3) = x^2 - 5x + 6\)

---

**Example:** Divide \(x^4 + 3x^3 - 2x + 2\) by \(x + 3\) and check the answer.

\[
x^2 + 0x - 2
\]

\[
x^4 + 3x^3 - 2x + 2
\]

Note: the first subtraction eliminated two terms from the dividend.

Therefore, the quotient skips a term.

Answer: \(x^2 - 2 + \frac{8}{x + 3}\)

Check: \((x + 3)(x^2 - 2) + 8 = x^4 + 3x^3 - 2x + 2\)