Polynomials

Multiplication of Polynomials

Multiplying a Monomial by a Monomial

Example 1: 
\((-2x^6)(3x^4) = -2 \cdot 3 \cdot x^6 \cdot x^4 = -6x^{6+4} = -6x^{10}\)

Example 2: 
\((10x^2y)(3x^9y^2)\)

\[10 \cdot 3 \cdot x^2 \cdot x^9 \cdot y^1 \cdot y^2 = 30x^{2+9}y^{1+2} = 30x^{11}y^3\]
Example 3: \((-4x^7y^0)(-9x^0yz^3) =\)

Write the answer before you click your mouse.

\[-4 \cdot -9 \cdot x^7 \cdot x^0 \cdot y^0 \cdot y^1 \cdot z^3\]

\[36x^{(7+0)}y^{(0+1)}z^3\]

\[36x^7yz^3\]

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**Multiplying a Monomial by a Polynomial**

Example 1: Distribute \(-2x\) thru parenthesis to each term.

\[-2x (x^2 - 3x + 9) = -2x(x^2) + (-2x)3x + (-2x)9 = -2x^3 + 6x^2 - 18x\]

Example 2: \(3a^2(-2a^3 + 8a - 10) =\)

\[3a^2(-2a^3) + 3a^2(8a) + 3a^2(-10) = -6a^5 + 24a^3 - 30a^2\]

Example 3: \((5x^2 - 4x + 6) (3x) =\)

\[5x^2 (3x) - 4x (3x) + 6 (3x) = 15x^3 - 12x^2 + 18x\]
Example 4: \((-3x^4 - 5x^2 + 1)(-3x^2)\)

\[= -3x^4(-3x^2) - 5x^2(-3x^2) + 1(-3x^2)\]

\[= 9x^6 + 15x^4 - 3x^2\]

Example 5: \(-2a(a^3 + a^2 - a + 4) =\)

\[-2a(a^3) + -2a(a^2) -2a(-a) + -2a(4) =\]

\[-2a^4 - 2a^3 + 2a^2 - 8a\]

**Multiplying a Binomial by a Binomial**

To multiply two binomials together we use an acronym called FOIL to help us remember the products.

- **F** stands for first. In the problem \((x + 4)(2x -5)\)
  - The first terms are \(x\) and \(2x\)
  - Their product is \(2x^2\)

- **O** stands for outside. \((x + 4)(2x -5)\)
  - The outside terms are \(x\) and \(-5\)
Multiplying a Binomial by a Binomial Continued

FOIL stands for First, Outside, Inside, Last.

I stands for inside. 
The inside terms are 4 and 2x
Their product is 8x

L stands for last. 
The last terms are 4 and -5
Their product is -20

Putting all the products together we get:
\((x + 4)(2x - 5) = 2x^2 - 5x + 8x - 20\)

Combining like terms the final answer is 
\(2x^2 + 3x - 20\)

Example 1: Multiply \((3y - 7)(5y - 6)\)

First \(3y(5y) = 15y^2\)
Outside \(3y(-6) = -18y\)
Inside \(-7 (5y) = -35y\)
Last \(-7 (-6) = +42\)

Answer is \(15y^2 - 18y - 35y + 42\)
Final Answer is \(15y^2 - 53y + 42\)
Do you see that each term of the first polynomial is multiplied by each term in the second polynomial?

Example 2: Multiply \((a + b)(c + d)\)

Distribute a thru \((c + d)\)
\[a(c + d) = ac + ad\]
First   Outside

Then distribute b
\[b(c + d) = bc + bd\]
Inside   Last

Final Answer is \(ac + ad + bc + bd\)

If you understand this basic premise: that each term of the first polynomial is multiplied by each term in the second polynomial, then it will be an easy transition to multiply polynomials containing more than two terms.

Example 3: Multiply \((2x + 3)(4x^2 - 3x - 2)\)

Because the second polynomial is not a binomial we cannot use FOIL. Instead multiply \(2x\) by \((4x^2 - 3x - 2)\) and then multiply \(3\) by \((4x^2 - 3x - 2)\).

The result is \(2x(4x^2 - 3x - 2) = 8x^3 - 6x^2 - 4x\) then
\[3(4x^2 - 3x - 2) = \\
12x^2 - 9x - 6\]

Now add down:
\[8x^3 + 6x^2 - 13x - 6\]