Division of Polynomials

**Dividing by a Monomial**

If the divisor only has one term, split the polynomial up into a fraction for each term.

\[
18x^4 - 24x^3 + 6x^2 - 12x = \frac{18x^4}{6x} - \frac{24x^3}{6x} + \frac{6x^2}{6x} - \frac{12x}{6x}
\]

Now reduce each fraction.

\[
= \frac{3x^3}{1} - \frac{4x^2}{1} + \frac{x}{1} - \frac{2}{1} = 3x^3 - 4x^2 + x - 2
\]

**Another Example**

If the divisor only has one term, split the polynomial up into a fraction for each term.

\[
6x^2y^3 - 2x^3 + 15x^2y - 12x + 3 = \frac{6x^2y^3}{6x^2y} - \frac{2x^3}{6x^2y} + \frac{15x^2y}{6x^2y} - \frac{12x}{6x^2y} + \frac{3}{6x^2y}
\]

Now reduce each fraction.

\[
= \frac{x^3y}{1} - \frac{x}{3y} + \frac{5}{2} - \frac{2}{xy} + \frac{1}{2x^2y}
\]
Another Example

If the divisor only has one term, split the polynomial up into a fraction for each term.

\[
\frac{18z^4 - 15z^3 + 6z^2 - 3z}{3z} = \frac{18z^4}{3z} - \frac{15z^3}{3z} + \frac{6z^2}{3z} - \frac{3z}{3z}
\]

Now reduce each fraction.

\[
\frac{6z^3}{3z} - \frac{5z^2}{3z} + \frac{2z}{3z} - \frac{1}{3z} = 6z^3 - 5z^2 + 2z - 1
\]

Dividing Polynomials Using Long Division

Long division of polynomials is similar to long division of whole numbers.

When you divide two polynomials you can check the answer using the following:

\[
\text{dividend} = (\text{quotient} \cdot \text{divisor}) + \text{remainder}
\]

The result is written in the form:

\[
\frac{\text{dividend}}{\text{divisor}} = \text{quotient} + \frac{\text{remainder}}{\text{divisor}}
\]

Example: Divide 10236 by 11 and check the answer.

1. \(11 \div 10236 = 9\)
2. \(9(1) = 09\)
3. \(102 - 99 = 03\)
4. \(10 + 3 = 13\)
5. \(3(11) = 33\)
6. \(33 - 33 = 00\)
7. \(11(0) = 00\)
8. \(0(1) = 00\)
9. \(00 - 0 = 00\)

Answer: \(930 + \frac{6}{11}\)

Check: \((930) \cdot 11 + \frac{6}{11} = 10236\) correct
Divide a Polynomial with 2 or more terms.

<table>
<thead>
<tr>
<th>Quotient</th>
<th>Divisor</th>
<th>Dividend</th>
</tr>
</thead>
</table>

Divide \((12 + X^2)\) by \((X + 3)\)

In a long division problem you must follow two set-up rules.

1) The dividend must be arranged in descending powers. Thus \(12 + X^2\) must be written as \(X^2 + 12\).

2) If there are any missing exponents in your dividend, you make space for them by adding a zero term.

Example 1: Divide \((X^2 + 5X + 12)\) by \((X + 3)\)

Set up the long division problem.

\[
\begin{array}{c|cc}
   & X^2 & + 5X + 12 \\
\hline
X+3 & X^2 & + 3X \\
\hline
   & X^2 & + 5X + 12 \\
   & -X^2 & - 3X \\
\hline
   & 2X & + 12 \\
\end{array}
\]

We are not finished yet so continue onto the next slide!

Divide the first term \(X^2\) by the first term in the divisor, \(X\). Write the result above \(5X\).

Multiply \(X\) by the divisor \(X + 3\) and write the answer below the dividend matching like terms as you go.

Subtract the bottom line by changing the signs of the bottom line you just wrote. When finished bring down the next term, which is \(12\).

\[
\begin{array}{c|cc}
   & X^2 & + 5X + 12 \\
\hline
X+3 & X^2 & + 3X \\
\hline
   & X^2 & + 5X + 12 \\
   & -X^2 & - 3X \\
\hline
   & 2X & + 12 \\
   & -2X & - 6 \\
\hline
   & 6 & \\
\end{array}
\]

The remainder is 6.

Divide the first term \(2X\) by the first term \(X\). The answer is 2.

Write 2 above the 12.

Multiply \(2(X + 3)\) = \(2X + 6\)

Write answer below \(2X + 12\).

Subtract by changing signs.

\[
\begin{array}{c|cc}
   & X^2 & + 5X + 12 \\
\hline
X+3 & X^2 & + 3X \\
\hline
   & X^2 & + 5X + 12 \\
   & -X^2 & - 3X \\
\hline
   & 2X & + 12 \\
   & -2X & - 6 \\
\hline
   & 6 & \\
\end{array}
\]

Write the final answer with the remainder in the form below.

\[
\begin{array}{c|cc}
   & X^2 + 5X + 12 \\
\hline
X+3 & X^2 + 3X \\
\hline
   & X^2 + 5X + 12 \\
   & -X^2 - 3X \\
\hline
   & 2X + 12 \\
   & -2X - 6 \\
\hline
   & 6 & \\
\end{array}
\]

The remainder is 6.
Example 2: Divide \((X^2 - 5)\) by \((X - 2)\)

Set up the long division problem.

\[
\begin{array}{c|cc}
\multicolumn{1}{r}{X - 2} & X^2 + 0X - 5 \\
\hline
X^2 - 2x \\
\hline
X^2 + 0X - 5 \\
\hline
-2x + 2x \\
\hline
+2x - 5 \\
\hline
\end{array}
\]

Divide the first term \(X^2\) by the first term in the divisor, \(X\). Write the result above \(0X\).

Multiply \(X\) by the divisor \(X - 2\) and write the answer below the dividend matching like terms as you go.

Subtract the bottom line by changing the signs of the bottom line you just wrote. When finished bring down the next term, which is \(-5\).

We are not finished yet so continue onto the next page!

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Example 2 (continued):

Divide the first term \(2X\) by the first term \(X\). The answer is \(2\). Write \(+2\) above the \(-5\).

Multiply \(2(X - 2) = 2X - 4\) Write answer below \(2X - 5\).

Subtract by changing signs. The remainder is \(-1\).

Write the final answer in the form on left.

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Example 3: Divide \(\frac{8X^3 - 1}{2X + 1}\)

\[
\begin{array}{c|ccc}
\multicolumn{1}{r}{2X + 1} & 8X^3 + 0X^2 + 0X - 1 \\
\hline
8X^3 + 0X^2 + 0X - 1 \\
\hline
4X^2 \\
\hline
8X^3 + 0X^2 + 0X - 1 \\
\hline
8X^3 + 4X^2 \\
\hline
\end{array}
\]

Step 1: Divide \(8X^3\) by \(2X\) Write answer before you click.

Step 2: Multiply \(4X^2\) by divisor. Write answer before you click.

We are not finished yet so continue onto the next page!
Example: Divide \( x^2 + 3x - 2 \) by \( x + 1 \) and check the answer.

\[
x + 1 \longdiv{x^2 + 3x - 2}
\]

1. \( x \) \( x^2 + 3x - 2 \)
2. \(-1\) \( -x - 1 \)
3. \( (x^2 + 3x) \) \( -2x \)
4. \( \frac{2x}{2} - 2 \)
5. \( 2(x + 1) - 2(x + 1) \)

Answer: \( x + 2 \) remainder \(-4\)

Check: \( (x + 2)(x + 1) + (-4) = x^2 + 3x - 2 \) correct
Example: Divide $4x + 2x^3 - 1$ by $2x - 2$ and check the answer.

\[ \begin{array}{c|cc}
   & x^2 + x + 3 \\
2x - 2 & 2x^3 + 0x^2 + 4x - 1 \\
   & 2x^3 + 2x^2 \\
   & 6x - 1 \\
   & 6x - 6 \\
   & 3 \\
\end{array} \]

Write the terms of the dividend in descending order.
Since there is no $x^0$ term in the dividend, add $1$ as a placeholder.
1. \( \frac{2x}{2x} = x\)
2. \( \frac{(2x)(2x)}{2x} = 2x^2 \)
3. \( \frac{x(2x^2 - 2x)}{2x} = x \)
4. \( \frac{2x^2 - x}{2x} = \frac{1}{2} \)
5. \( \frac{x(2x - 2)}{2x} = 2x - 2 \)
6. \( \frac{(2x^3 + 4x) - (2x^2 - 2x)}{x} = 6x \)
7. \( \frac{6x}{2x} = 3 \)
8. \( 3(2x - 2) = 6x - 6 \)
9. \( 6x - 1 - (6x - 6) = 5 = \text{remainder} \)

Answer: $x^2 + x + 3 + \frac{5}{2x - 2}$
Check: $(x^2 + x + 3)(2x - 2) + 5 = 4x + 2x^3 - 1$

Example: Divide $x^2 - 5x + 6$ by $x - 2$.

\[ \begin{array}{c|cc}
   & x - 3 \\
x - 2 & x^2 - 5x + 6 \\
   & x^2 - 2x \\
   & -3x + 6 \\
   & -3x + 6 \\
   & 0 \\
\end{array} \]

Answer: $x - 3$ with no remainder.
Check: $(x - 2)(x - 3) = x^2 - 5x + 6$

Example: Divide $x^4 + 3x^2 - 2x + 2$ by $x + 3$ and check the answer.

\[ \begin{array}{c|cc}
   & x^2 + 0x - 2 \\
\begin{array}{c}x + 3 \\
   \end{array} & x^4 + 3x^2 - 2x + 2 \\
   & x^4 + 3x^2 \\
   & \downarrow \ \\
   & -0x^2 - 2x + 2 \\
   & -2x - 6 \\
   & 8 \\
\end{array} \]

Note: the first subtraction eliminated two terms from the dividend.
Therefore, the quotient skips a term.

Answer: $x^2 - 2 + \frac{8}{x + 3}$
Check: $(x + 3)(x^2 - 2) + 8 = x^4 + 3x^2 - 2x + 2$