Completing the Square

Objectives
- Solve equations of the form \( x^2 = \text{some number} \).
- Solve equations of the form \((ax + b)^2 = \text{a number}\).

Factoring
- Before today the only way we had for solving quadratics was to factor.
  \[ x^2 - 2x - 15 = 0 \]
  \[ (x + 3)(x - 5) = 0 \]
  \[ x + 3 = 0 \text{ or } x - 5 = 0 \]
  \[ x = -3 \text{ or } x = 5 \]
- Zero-factor property
- Solution Set
Factoring

\[ x^2 = 9 \]
\[ x^2 - 9 = 0 \]
\[ (x + 3)(x - 3) = 0 \]
\[ x + 3 = 0 \text{ or } x - 3 = 0 \]
\[ x = -3 \text{ or } x = 3 \]

Solution Set

Another Way to Solve Quadratics
Square Root Property

\[ x^2 = 9 \]
\[ \sqrt{x^2} = \sqrt{9} \]
\[ x = 3 \]
\[ x^2 = 9 \]
\[ \sqrt{x^2} = \pm \sqrt{9} \]
\[ x = \pm 3 \]

Recall that we know the solution set is 3 or -3
When you introduce the radical you must use + and - signs.

Solve each equation.
Write radicals in simplified form.

\[ k^2 = 49 \]
\[ k = \pm \sqrt{49} \quad \text{Square Root Property} \]
\[ k = \pm 7 \]
\[ k = +7 \text{ or } -7 \]

Solution Set
Solve each equation.
Write radicals in simplified form.

\[ b^2 = 11 \]
\[ b = \pm \sqrt{11} \quad \text{Square Root Property} \]
Radical will not simplify.
\[ b = -\sqrt{11} \quad \text{or} \quad \sqrt{11} \quad \text{Solution Set} \]

Solve each equation.
Write radicals in simplified form.

\[ c^2 = 12 \]
\[ c = \pm \sqrt{12} \quad \text{Square Root Property} \]
\[ c = \pm 2\sqrt{3} \]
\[ c = -2\sqrt{3} \quad \text{or} \quad 2\sqrt{3} \quad \text{Solution Set} \]

Solve each equation.
Write radicals in simplified form.

\[ x^2 = -9 \]
\[ x = \pm \sqrt{-9} \quad \text{Square Root Property} \]
\[ x = \pm 3i \quad \text{Yikes! A negative inside a radical.} \]
Solve each equation. Write radicals in simplified form.

\[(m + 2)^2 = 36\]

\[m + 2 = \pm \sqrt{36}\]

\[m + 2 = \pm 6\]

\[m = -8 \quad \text{or} \quad 4\]

\[
\begin{align*}
2m + 6 &= 36 \\
2m &= 30 \\
m &= 15
\end{align*}
\]

\[
\begin{align*}
m + 2 &= -6 \\
-2 &= -2 \\
m &= -8
\end{align*}
\]

\[
\begin{align*}
m + 2 &= 6 \\
-2 &= -2 \\
m &= 4
\end{align*}
\]

Notice the solutions are conjugate factors.

Solving Quadratic Equations by Completing the Square

\[x^2 - 2x - 15 = 0\]

\[(x + 3)(x - 5) = 0\]

\[x + 3 = 0\]

or \[x - 5 = 0\]

\[x = -3 \quad \text{or} \quad x = 5\]
### Solving Quadratic Equations by Completing the Square

**Example:**

1. **Equation:** \( x^2 - 2x - 15 = 0 \)
2. **Completed Square:** \( x^2 - 2x + 1 = 15 + 1 \)
3. **Simplified:** \( (x - 1)^2 = 16 \)
4. **Solutions:** \( x = 1 \pm 4 \)
5. **Final Answer:** \( x = \{-3, 5\} \)

### Completing the Square

**Steps:**
1. Make the coefficient of the squared term = 1.
2. Move all variables to one side and constants to the other.
3. Take half of the coefficient of the x term and square it. Then add to both sides of the equation.
4. Factor the left hand side and simplify the right.
5. Root and solve.

**Example:**

1. **Equation:** \( 3x^2 + 5x - 2 = 0 \)
2. **Simplified:** \( x^2 + \frac{5}{3}x - \frac{2}{3} = 0 \)
3. **Completed Square:** \( x^2 + \frac{5}{3}x + \frac{25}{36} = \frac{29}{36} \)
4. **Solutions:** \( x + \frac{5}{6} = \pm \frac{\sqrt{29}}{6} \)
5. **Final Answer:** \( x = \{-2, \frac{1}{3}\} \)

### Completing the Square continued . . .

1. **Equation:** \( 3x^2 + 5x - 2 = 0 \)
2. **Simplified:** \( x^2 + \frac{5}{3}x - \frac{2}{3} = 0 \)
3. **Completed Square:** \( x^2 + \frac{5}{3}x + \frac{25}{36} = \frac{29}{36} \)
4. **Solutions:** \( x + \frac{5}{6} = \pm \frac{\sqrt{29}}{6} \)
5. **Final Answer:** \( x = \{-2, \frac{1}{3}\} \)

**Verification:**

- \( x = -\frac{5}{6} - \frac{7}{6} = -\frac{12}{6} = -2 \)
- \( x = \frac{5}{6} + \frac{7}{6} = \frac{1}{3} \)
Quadratic Formula

\[
\frac{ax^2 + bx + c}{a} = \frac{b}{a}x + \frac{c}{a} = 0
\]

\[
\left( \frac{b}{2a} \right)^2 - \frac{b^2}{4a^2} = \frac{b^2 - 4ac}{4a^2}
\]

\[
\left( x + \frac{b}{2a} \right)^2 = \frac{b^2 - 4ac}{4a^2}
\]

\[
x + \frac{b}{2a} = \pm \sqrt{\frac{b^2 - 4ac}{4a^2}}
\]

\[
x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}
\]