Graphs of Functions

Increasing, Decreasing, and Constant Functions

A function is increasing on an interval if for any $x_1$ and $x_2$ in the interval, where $x_1 < x_2$, then $f(x_1) < f(x_2)$.

A function is decreasing on an interval if for any $x_1$ and $x_2$ in the interval, where $x_1 < x_2$, then $f(x_1) > f(x_2)$.

A function is constant on an interval if for any $x_1$ and $x_2$ in the interval, where $x_1 < x_2$, then $f(x_1) = f(x_2)$.

Solution

a. The function is decreasing on the interval $(-\infty, 0)$, increasing on the interval $(0, 2)$, and decreasing on the interval $(2, \infty)$.
Figure Example cont.

Describe the increasing, decreasing, or constant behavior of each function whose graph is shown.

a. 

b. Although the function's equations are not given, the graph indicates that the function is defined in two pieces. The part of the graph to the left of the y-axis shows that the function is constant on the interval (-∞, 0). The part to the right of the y-axis shows that the function is increasing on the interval (0, ∞).

Definitions of Relative Maximum and Relative Minimum

1. A function value \( f(a) \) is a **relative maximum** of \( f \) if there exists an open interval about \( a \) such that \( f(x) > f(x) \) for all \( x \) in the open interval.

2. A function value \( f(b) \) is a **relative minimum** of \( f \) if there exists an open interval about \( b \) such that \( f(b) < f(x) \) for all \( x \) in the open interval.

The Average Rate of Change of a Function

- Let \((x_1, f(x_1))\) and \((x_2, f(x_2))\) be distinct points on the graph of a function \( f \). The **average rate of change of \( f \)** from \( x_1 \) to \( x_2 \) is

\[
\frac{f(x_2) - f(x_1)}{x_2 - x_1}
\]
Graphs of Functions