Linear Equations

Terms Involving Equations

An equation consists of two algebraic expressions joined by an equal sign.

\[ 3x - 1 = 2 \]
\[ 3x = 3 \]
\[ x = 1 \]

1 is a solution or root of the equation

Definition of a Linear Equation

- A linear equation in one variable \( x \) is an equation that can be written in the form
- \( ax + b = 0 \)
- where \( a \) and \( b \) are real numbers and \( a \neq 0 \).
Generating Equivalent Equations

An equation can be transformed into an equivalent equation by one or more of the following operations.

1. Simplify an expression by removing grouping symbols and combining like terms.
   
   Example
   
   \[
   \begin{align*}
   3(x - 6) &= 6x - x \\
   3x - 18 &= 5x \\
   -18 &= 2x \\
   -9 &= x
   \end{align*}
   \]

2. Add (or subtract) the same real number or variable expression on both sides of the equation.

3. Multiply (or divide) on both sides of the equation by the same nonzero quantity.

4. Interchange the two sides of the equation.

Solving a Linear Equation

- Simplify the algebraic expression on each side.
- Collect all the variable terms on one side and all the constant terms on the other side.
- Isolate the variable and solve.
- Check the proposed solution in the original equation.

Text Example

Solve the equation: \(2(x - 3) - 17 = 13 - 3(x + 2)\).

**Solution**

**Step 1** Simplify the algebraic expression on each side.

\[
\begin{align*}
2(x - 3) &= 17 - 3(x + 2) \\
2x - 6 &= 17 - 3x - 6 \\
2x - 3x &= 10 \\
-x &= 10 \\
x &= -10
\end{align*}
\]
Solution

Step 2 Collect variable terms on one side and constant terms on the other side. We will collect variable terms on the left by adding $3x$ to both sides. We will collect the numbers on the right by adding 23 to both sides.

\[
2x - 23 + 3x = -3x + 7 + 3x \\
5x - 23 = 7 \\
5x - 23 + 23 = 7 + 23 \\
5x = 30
\]

Add $3x$ to both sides.
Simplify.
Add 23 to both sides.
Simplify.

Step 3 Isolate the variable and solve. We isolate the variable by dividing both sides by 5.

\[
\frac{5x}{5} = \frac{30}{5} \\
x = 6
\]

Divide both sides by 5.
Simplify.

Step 4 Check the proposed solution in the original equation. Substitute 6 for $x$ in the original equation.

\[
2(x - 3) - 17 = 13 - 3(x + 2) \\
2(6 - 3) - 17 = 13 - 3(6 + 2) \\
2(3) - 17 = 13 - 3(8) \\
6 - 17 = 13 - 24 \\
-11 = -11
\]

This is the original equation.
Substitute 6 for $x$.
Simplify inside parentheses.
Multiply.
This true statement indicates that 6 is the solution.

The solution set is \{6\}.  

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Text Example

Solve the equation: $2(x - 3) - 17 = 13 - 3(x + 2)$.

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Types of Equations

- **Identity**: An equation that is true for all real numbers.
- **Conditional**: An equation that is true for at least one real number.
- **Inconsistent**: An equation that is not true for any real number.

Example

Determine whether the equation $3(x - 1) = 3x + 5$ is an identity, a conditional equation, or an inconsistent equation.

**Solution**  
To find out, solve the equation.

\[
3(x - 1) = 3x + 5 \\
3x - 3 = 3x + 5 \\\n-3 = 5
\]

This equation is inconsistent.

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