SECTION 5.2 Right Triangle Trigonometry

Objectives

1. Use right triangles to evaluate trigonometric functions.
2. Find function values for $30^\circ \left( \frac{\pi}{6} \right)$, $45^\circ \left( \frac{\pi}{4} \right)$, and $60^\circ \left( \frac{\pi}{3} \right)$.
3. Recognize and use fundamental identities.
4. Use equal cofunctions of complements.
5. Evaluate trigonometric functions with a calculator.
6. Use right triangle trigonometry to solve applied problems.

Mountain climbers have forever been fascinated by reaching the top of Mount Everest, sometimes with tragic results. The mountain, on Asia’s Tibet-Nepal border, is Earth’s highest, peaking at an incredible 29,035 feet. The heights of mountains can be found using trigonometry. The word trigonometry means measurement of triangles. Trigonometry is used in navigation, building, and engineering. For centuries, Muslims used trigonometry and the stars to navigate across the Arabian desert to Mecca, the birthplace of the prophet Muhammad, the founder of Islam. The ancient Greeks used trigonometry to record the locations of thousands of stars and worked out the motion of the Moon relative to Earth. Today, trigonometry is used to study the structure of DNA, the master molecule that determines how we grow from a single cell to a complex, fully developed adult.

The Six Trigonometric Functions

We begin the study of trigonometry by defining six functions, the six trigonometric functions. The inputs for these functions are measures of acute angles in right triangles. The outputs are the ratios of the lengths of the sides of right triangles. Figure 5.14 shows a right triangle with one of its acute angles labeled $\theta$. The side opposite the right angle is known as the hypotenuse. The other sides of the triangle are described by their position relative to the acute angle $\theta$. One side is opposite $\theta$ and one is adjacent to $\theta$.

The trigonometric functions have names that are words, rather than single letters such as $f$, $g$, and $h$. For example, the sine of $\theta$ is the length of the side opposite $\theta$ divided by the length of the hypotenuse:

$$\sin \theta = \frac{\text{length of side opposite } \theta}{\text{length of hypotenuse}}.$$ 

The ratio of lengths depends on angle $\theta$ and thus is a function of $\theta$. The expression $\sin \theta$ really means $\sin(\theta)$, where sine is the name of the function and $\theta$, the measure of an acute angle, is an input.
Here are the names of the six trigonometric functions, along with their abbreviations:

<table>
<thead>
<tr>
<th>Name</th>
<th>Abbreviation</th>
<th>Name</th>
<th>Abbreviation</th>
</tr>
</thead>
<tbody>
<tr>
<td>sine</td>
<td>sin</td>
<td>cosecant</td>
<td>csc</td>
</tr>
<tr>
<td>cosine</td>
<td>cos</td>
<td>secant</td>
<td>sec</td>
</tr>
<tr>
<td>tangent</td>
<td>tan</td>
<td>cotangent</td>
<td>cot</td>
</tr>
</tbody>
</table>

Now, let $\theta$ be an acute angle in a right triangle, shown in Figure 5.15. The length of the side opposite $\theta$ is $a$, the length of the side adjacent to $\theta$ is $b$, and the length of the hypotenuse is $c$.

![Figure 5.15](Image)

**Right Triangle Definitions of Trigonometric Functions**

See Figure 5.15. The six trigonometric functions of the acute angle $\theta$ are defined as follows:

\[
\sin \theta = \frac{\text{length of side opposite angle } \theta}{\text{length of hypotenuse}} = \frac{a}{c} \quad \csc \theta = \frac{\text{length of hypotenuse}}{\text{length of side opposite angle } \theta} = \frac{c}{a}
\]
\[
\cos \theta = \frac{\text{length of side adjacent to angle } \theta}{\text{length of hypotenuse}} = \frac{b}{c} \quad \sec \theta = \frac{\text{length of hypotenuse}}{\text{length of side adjacent to angle } \theta} = \frac{c}{b}
\]
\[
\tan \theta = \frac{\text{length of side opposite angle } \theta}{\text{length of side adjacent to angle } \theta} = \frac{a}{b} \quad \cot \theta = \frac{\text{length of side adjacent to angle } \theta}{\text{length of side opposite angle } \theta} = \frac{b}{a}
\]

Each of the trigonometric functions of the acute angle $\theta$ is positive. Observe that the functions in the second column in the box are the reciprocals of the corresponding functions in the first column.

Figure 5.16 on the next page shows four right triangles of varying sizes. In each of the triangles, $\theta$ is the same acute angle, measuring approximately 56.3°. All four of these similar triangles have the same shape and the lengths of corresponding sides are in the same ratio. In each triangle, the tangent function has the same value: $\tan \theta = \frac{3}{4}$. 

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The functions in the second column are reciprocals of those in the first column. You can obtain their values by exchanging the numerator and denominator of the corresponding ratios in the first column.

Study Tip

In general, the trigonometric function values of \( \theta \) depend only on the size of angle \( \theta \), and not on the size of the triangle.

**Example 1** Evaluating Trigonometric Functions

Find the value of each of the six trigonometric functions of \( \theta \) in Figure 5.17.

**Solution** We need to find the values of the six trigonometric functions of \( \theta \). However, we must know the lengths of all three sides of the triangle (a, b, and c) to evaluate all six functions. The values of a and b are given. We can use the Pythagorean Theorem, \( c^2 = a^2 + b^2 \), to find c.

\[
a = 5 \quad b = 12
\]

\[
c^2 = a^2 + b^2 = 5^2 + 12^2 = 25 + 144 = 169
\]

\[
c = \sqrt{169} = 13
\]

Now that we know the lengths of the three sides of the triangle, we apply the definitions of the six trigonometric functions of \( \theta \). Referring to these lengths as opposite, adjacent, and hypotenuse, we have

\[
\sin \theta = \frac{\text{opposite}}{\text{hypotenuse}} = \frac{5}{13}
\]

\[
\csc \theta = \frac{\text{hypotenuse}}{\text{opposite}} = \frac{13}{5}
\]

\[
\cos \theta = \frac{\text{adjacent}}{\text{hypotenuse}} = \frac{12}{13}
\]

\[
\sec \theta = \frac{\text{hypotenuse}}{\text{adjacent}} = \frac{13}{12}
\]

\[
\tan \theta = \frac{\text{opposite}}{\text{adjacent}} = \frac{5}{12}
\]

\[
\cot \theta = \frac{\text{adjacent}}{\text{opposite}} = \frac{12}{5}.
\]

**Check Point** Find the value of each of the six trigonometric functions of \( \theta \) in the figure.
Function Values for Some Special Angles

A 45°, or \(\frac{\pi}{4}\) radian, angle occurs frequently in trigonometry. How do we find the values of the trigonometric functions of 45°? We construct a right triangle with a 45° angle, shown in Figure 5.18. The triangle actually has two 45° angles. Thus, the triangle is isosceles—that is, it has two sides of the same length. Assume that each leg of the triangle has a length equal to 1. We can find the length of the hypotenuse using the Pythagorean Theorem.

\[
(\text{length of hypotenuse})^2 = 1^2 + 1^2 = 2
\]

length of hypotenuse = \(\sqrt{2}\)

With Figure 5.18, we can determine the trigonometric function values for 45°.

Example 2 Evaluating Trigonometric Functions of 45°

Use Figure 5.18 to find \(\sin 45^\circ\), \(\cos 45^\circ\), and \(\tan 45^\circ\).

Solution We apply the definitions of these three trigonometric functions.

\[
\sin 45^\circ = \frac{\text{length of side opposite } 45^\circ}{\text{length of hypotenuse}} = \frac{1}{\sqrt{2}}
\]

\[
\cos 45^\circ = \frac{\text{length of side adjacent to } 45^\circ}{\text{length of hypotenuse}} = \frac{1}{\sqrt{2}}
\]

\[
\tan 45^\circ = \frac{\text{length of side opposite } 45^\circ}{\text{length of side adjacent to } 45^\circ} = \frac{1}{1} = 1
\]

Check Point 2 Use Figure 5.18 to find \(\csc 45^\circ\), \(\sec 45^\circ\), and \(\cot 45^\circ\).

When you worked Check Point 2, did you actually use Figure 5.18 or did you use reciprocals to find the values?

\[
\csc 45^\circ = \sqrt{2} \quad \sec 45^\circ = \sqrt{2} \quad \cot 45^\circ = 1
\]

Take the reciprocal of \(\sin 45^\circ = \frac{1}{\sqrt{2}}\).

Take the reciprocal of \(\cos 45^\circ = \frac{1}{\sqrt{2}}\).

Take the reciprocal of \(\tan 45^\circ = \frac{1}{1}\).

We found that \(\sin 45^\circ = \frac{1}{\sqrt{2}}\) and \(\cos 45^\circ = \frac{1}{\sqrt{2}}\). This value is often expressed by rationalizing the denominator:

\[
\frac{1}{\sqrt{2}} = \frac{1}{\sqrt{2}} \cdot \frac{\sqrt{2}}{\sqrt{2}} = \frac{\sqrt{2}}{2}.
\]

We are multiplying by 1 and not changing the value of \(\frac{1}{\sqrt{2}}\).

Thus, \(\sin 45^\circ = \frac{\sqrt{2}}{2}\) and \(\cos 45^\circ = \frac{\sqrt{2}}{2}\).
Two other angles that occur frequently in trigonometry are $30^\circ$, or $\frac{\pi}{6}$ radian, and $60^\circ$, or $\frac{\pi}{3}$ radian, angles. We can find the values of the trigonometric functions of $30^\circ$ and $60^\circ$ by using a right triangle. To form this right triangle, draw an equilateral triangle—that is, a triangle with all sides the same length. Assume that each side has a length equal to 2. Now take half of the equilateral triangle. We obtain the right triangle in Figure 5.19. This right triangle has a hypotenuse of length 2 and a leg of length 1. The other leg has length $a$, which can be found using the Pythagorean Theorem.

$$a^2 + 1^2 = 2^2$$
$$a^2 + 1 = 4$$
$$a^2 = 3$$
$$a = \sqrt{3}$$

With the right triangle in Figure 5.19, we can determine the trigonometric functions for $30^\circ$ and $60^\circ$.

**Example 3 Evaluating Trigonometric Functions of $30^\circ$ and $60^\circ$**

Use Figure 5.19 to find $\sin 60^\circ$, $\cos 60^\circ$, $\sin 30^\circ$, and $\cos 30^\circ$.

**Solution**

We begin with $60^\circ$. Use the angle on the lower left in Figure 5.19.

$$\sin 60^\circ = \frac{\text{length of side opposite } 60^\circ}{\text{length of hypotenuse}} = \frac{\sqrt{3}}{2}$$

$$\cos 60^\circ = \frac{\text{length of side adjacent to } 60^\circ}{\text{length of hypotenuse}} = \frac{1}{2}$$

To find $\sin 30^\circ$ and $\cos 30^\circ$, use the angle on the upper right in Figure 5.19.

$$\sin 30^\circ = \frac{\text{length of side opposite } 30^\circ}{\text{length of hypotenuse}} = \frac{1}{2}$$

$$\cos 30^\circ = \frac{\text{length of side adjacent to } 30^\circ}{\text{length of hypotenuse}} = \frac{\sqrt{3}}{2}$$

Use Figure 5.19 to find $\tan 60^\circ$ and $\tan 30^\circ$. If a radical appears in a denominator, rationalize the denominator.

Because we will often use the function values of $30^\circ$, $45^\circ$, and $60^\circ$, you should learn to construct the right triangles shown in Figures 5.18 and 5.19. With sufficient practice, you will memorize the values in the box on the next page.
Recognize and use fundamental identities.

Many relationships exist among the six trigonometric functions. These relationships are described using *trigonometric identities*. For example, $\csc \theta$ is defined as the reciprocal of $\sin \theta$. This relationship can be expressed by the identity

$$
\csc \theta = \frac{1}{\sin \theta}.
$$

This identity is one of six *reciprocal identities*.

**Reciprocal Identities**

- $\sin \theta = \frac{1}{\csc \theta}$
- $\cos \theta = \frac{1}{\sec \theta}$
- $\tan \theta = \frac{1}{\cot \theta}$
- $\csc \theta = \frac{1}{\sin \theta}$
- $\sec \theta = \frac{1}{\cos \theta}$
- $\cot \theta = \frac{1}{\tan \theta}$

Two other relationships that follow from the definitions of the trigonometric functions are called the *quotient identities*.

**Quotient Identities**

- $\tan \theta = \frac{\sin \theta}{\cos \theta}$
- $\cot \theta = \frac{\cos \theta}{\sin \theta}$

If $\sin \theta$ and $\cos \theta$ are known, a quotient identity and three reciprocal identities make it possible to find the value of each of the four remaining trigonometric functions.
EXAMPLE 4 Using Quotient and Reciprocal Identities

Given \( \sin \theta = \frac{1}{2} \) and \( \cos \theta = \frac{\sqrt{3}}{2} \), find the value of each of the four remaining trigonometric functions.

**Solution** We can find \( \tan \theta \) by using the quotient identity that describes \( \tan \theta \) as the quotient of \( \sin \theta \) and \( \cos \theta \).

\[
\tan \theta = \frac{\sin \theta}{\cos \theta} = \frac{\frac{1}{2}}{\frac{\sqrt{3}}{2}} = \frac{1}{2} \cdot \frac{2}{\sqrt{3}} = \frac{1}{\sqrt{3}} \cdot \frac{\sqrt{3}}{\sqrt{3}} = \frac{\sqrt{3}}{3}
\]

We use the reciprocal identities to find the value of each of the remaining three functions:

\[
csc \theta = \frac{1}{\sin \theta} = \frac{1}{\frac{1}{2}} = 2
\]

\[
sec \theta = \frac{1}{\cos \theta} = \frac{1}{\frac{\sqrt{3}}{2}} = \frac{2}{\sqrt{3}} \cdot \frac{\sqrt{3}}{\sqrt{3}} = \frac{2\sqrt{3}}{3}
\]

\[
cot \theta = \frac{1}{\tan \theta} = \frac{1}{\frac{1}{\sqrt{3}}} = \sqrt{3}
\]

We found \( \tan \theta = \frac{\sqrt{3}}{3} \). We could use \( \tan \theta = \frac{\sqrt{3}}{3} \), but then we would have to rationalize the denominator.

Other relationships among trigonometric functions follow from the Pythagorean Theorem. Using Figure 5.20, the Pythagorean Theorem states that

\[ a^2 + b^2 = c^2. \]

To obtain ratios that correspond to trigonometric functions, divide both sides of this equation by \( c^2 \).

\[
\frac{a^2}{c^2} + \frac{b^2}{c^2} = 1 \quad \text{or} \quad \left( \frac{a}{c} \right)^2 + \left( \frac{b}{c} \right)^2 = 1
\]

In Figure 5.20, \( \sin \theta = \frac{a}{c} \), so this is \( (\sin \theta)^2 \). In Figure 5.20, \( \cos \theta = \frac{b}{c} \), so this is \( (\cos \theta)^2 \).

Based on the observations in the voice balloons, we see that

\[
(\sin \theta)^2 + (\cos \theta)^2 = 1.
\]
We will use the notation $\sin^2 \theta$ for $(\sin \theta)^2$ and $\cos^2 \theta$ for $(\cos \theta)^2$. With this notation, we can write the identity as

$$\sin^2 \theta + \cos^2 \theta = 1.$$  

Two additional identities can be obtained from $a^2 + b^2 = c^2$ by dividing both sides by $b^2$ and $a^2$, respectively. The three identities are called the Pythagorean identities.

**Pythagorean Identities**

$$\sin^2 \theta + \cos^2 \theta = 1 \quad 1 + \tan^2 \theta = \sec^2 \theta \quad 1 + \cot^2 \theta = \csc^2 \theta$$

### Example 5 Using a Pythagorean Identity

Given that $\sin \theta = \frac{3}{5}$ and $\theta$ is an acute angle, find the value of $\cos \theta$ using a trigonometric identity.

**Solution**  

We can find the value of $\cos \theta$ by using the Pythagorean identity

$$\sin^2 \theta + \cos^2 \theta = 1.$$  

We are given that $\sin \theta = \frac{3}{5}$.  

$$\left(\frac{3}{5}\right)^2 + \cos^2 \theta = 1$$  

Square: $\left(\frac{3}{5}\right)^2 = \frac{9}{25} = \frac{9}{25}$.

$$\frac{9}{25} + \cos^2 \theta = 1$$  

Subtract $\frac{9}{25}$ from both sides.

$$\cos^2 \theta = 1 - \frac{9}{25}$$  

Simplify: $1 - \frac{9}{25} = \frac{25}{25} - \frac{9}{25} = \frac{16}{25}$.

$$\cos^2 \theta = \frac{16}{25}$$  

Because $\theta$ is an acute angle, $\cos \theta$ is positive.

Thus, $\cos \theta = \frac{4}{5}$.

### Check Point

Given that $\sin \theta = \frac{1}{3}$ and $\theta$ is an acute angle, find the value of $\cos \theta$ using a trigonometric identity.

### Trigonometric Functions and Complements

Another relationship among trigonometric functions is based on angles that are complements. Refer to Figure 5.21. Because the sum of the angles of any triangle is $180^\circ$, in a right triangle the sum of the acute angles is $90^\circ$. Thus, the acute angles are complements. If the degree measure of one acute angle is $\theta$, then the degree measure of the other acute angle is $(90^\circ - \theta)$. This angle is shown on the upper right in Figure 5.21.

Let’s use Figure 5.21 to compare $\sin \theta$ and $\cos(90^\circ - \theta)$.

$$\sin \theta = \frac{\text{length of side opposite } \theta}{\text{length of hypotenuse}} = \frac{a}{c}$$  

$$\cos(90^\circ - \theta) = \frac{\text{length of side adjacent to } (90^\circ - \theta)}{\text{length of hypotenuse}} = \frac{a}{c}$$

Thus, $\sin \theta = \cos(90^\circ - \theta)$. If two angles are complements, the sine of one equals the cosine of the other. Because of this relationship, the sine and cosine are called equal cofunctions.
cofunctions of each other. The name cosine is a shortened form of the phrase complement's sine.

Any pair of trigonometric functions \( f \) and \( g \) for which

\[
f(\theta) = g(90^\circ - \theta) \quad \text{and} \quad g(\theta) = f(90^\circ - \theta)
\]

are called cofunctions. Using Figure 5.21, we can show that the tangent and cotangent are also cofunctions of each other. So are the secant and cosecant.

**Cofunction Identities**

The value of a trigonometric function of \( \theta \) is equal to the cofunction of the complement of \( \theta \).

\[
\sin \theta = \cos(90^\circ - \theta) \quad \cos \theta = \sin(90^\circ - \theta) \\
\tan \theta = \cot(90^\circ - \theta) \quad \cot \theta = \tan(90^\circ - \theta) \\
\sec \theta = \csc(90^\circ - \theta) \quad \csc \theta = \sec(90^\circ - \theta)
\]

If \( \theta \) is in radians, replace \( 90^\circ \) with \( \frac{\pi}{2} \).

**Example 6**

Find a cofunction with the same value as the given expression:

\[
a. \sin 72^\circ \quad b. \csc \frac{\pi}{3}
\]

**Solution** Because the value of a trigonometric function of \( \theta \) is equal to the cofunction of the complement of \( \theta \), we need to find the complement of each angle. We do this by subtracting the angle's measure from \( 90^\circ \) or its radian equivalent, \( \frac{\pi}{2} \).

\[
a. \sin 72^\circ = \cos(90^\circ - 72^\circ) = \cos 18^\circ
\]

\[
b. \csc \frac{\pi}{3} = \sec \left( \frac{\pi}{2} - \frac{\pi}{3} \right) = \sec \left( \frac{3\pi}{6} - \frac{2\pi}{6} \right) = \sec \frac{\pi}{6}
\]

Evaluate trigonometric functions with a calculator.

**Using a Calculator to Evaluate Trigonometric Functions**

The values of the trigonometric functions obtained with the special triangles are exact values. For most angles other than \( 30^\circ, 45^\circ, \) and \( 60^\circ \), we approximate the value of each of the trigonometric functions using a calculator. The first step is
to set the calculator to the correct mode, degrees or radians, depending on how the acute angle is measured.

Most calculators have keys marked \([\sin], [\cos],\) and \([\tan]\). For example, to find the value of \(\sin 30^\circ\), set the calculator to the degree mode and enter 30 \([\sin]\) on most scientific calculators and \([\sin] 30 [\text{ENTER}]\) on most graphing calculators. Consult the manual for your calculator.

To evaluate the cosecant, secant, and cotangent functions, use the key for the respective reciprocal function, \([\sin], [\cos],\) or \([\tan]\), and then use the reciprocal key. The reciprocal key is \([1/x]\) on many scientific calculators and \([x^{-1}]\) on many graphing calculators. For example, we can evaluate \(\sec \frac{\pi}{12}\) using the following reciprocal relationship:

\[
\sec \frac{\pi}{12} = \frac{1}{\cos \frac{\pi}{12}}.
\]

Using the radian mode, enter one of the following keystroke sequences:

**Many Scientific Calculators**

\[
\pi \div 12 = \cos \frac{1}{x}
\]

**Many Graphing Calculators**

\[
\cos \left( \frac{\pi}{12} \right) \div \cos \left( \frac{1}{x} \right) \text{ ENTER}
\]

Rounding the display to four decimal places, we obtain \(\sec \frac{\pi}{12} = 1.0353\).

**EXAMPLE 7 Evaluating Trigonometric Functions with a Calculator**

Use a calculator to find the value to four decimal places:

- **a.** \(\cos 48.2^\circ\)
- **b.** \(\cot 1.2\).

**Solution**

**Scientific Calculator Solution**

<table>
<thead>
<tr>
<th>Function</th>
<th>Mode</th>
<th>Keystrokes</th>
<th>Display, rounded to four decimal places</th>
</tr>
</thead>
<tbody>
<tr>
<td>a. (\cos 48.2^\circ)</td>
<td>D degree</td>
<td>48.2 (\cos)</td>
<td>0.6665</td>
</tr>
<tr>
<td>b. (\cot 1.2)</td>
<td>Radian</td>
<td>1.2 (\tan \frac{1}{x})</td>
<td>0.3888</td>
</tr>
</tbody>
</table>

**Graphing Calculator Solution**

<table>
<thead>
<tr>
<th>Function</th>
<th>Mode</th>
<th>Keystrokes</th>
<th>Display, rounded to four decimal places</th>
</tr>
</thead>
<tbody>
<tr>
<td>a. (\cos 48.2^\circ)</td>
<td>D degree</td>
<td>(\cos 48.2 \text{ ENTER})</td>
<td>0.6665</td>
</tr>
<tr>
<td>b. (\cot 1.2)</td>
<td>Radian</td>
<td>(\frac{\tan 1.2}{x} \text{ ENTER})</td>
<td>0.3888</td>
</tr>
</tbody>
</table>
Use a calculator to find the value to four decimal places:

\[
\begin{align*}
\text{a. } & \sin 72.8^\circ \\
\text{b. } & \csc 1.5.
\end{align*}
\]

Applications

Many applications of right triangle trigonometry involve the angle made with an imaginary horizontal line. As shown in Figure 5.22, an angle formed by a horizontal line and the line of sight to an object that is above the horizontal line is called the angle of elevation. The angle formed by a horizontal line and the line of sight to an object that is below the horizontal line is called the angle of depression. Transits and sextants are instruments used to measure such angles.

EXAMPLE 8 Problem Solving Using an Angle of Elevation

Sighting the top of a building, a surveyor measured the angle of elevation to be 22°. The transit is 5 feet above the ground and 300 feet from the building. Find the building’s height.

Solution The situation is illustrated in Figure 5.23. Let \( a \) be the height of the portion of the building that lies above the transit. The height of the building is the transit’s height, 5 feet, plus \( a \). Thus, we need to identify a trigonometric function that will make it possible to find \( a \). In terms of the 22° angle, we are looking for the side opposite the angle. The transit is 300 feet from the building, so the side adjacent to the 22° angle is 300 feet. Because we have a known angle, an unknown opposite side, and a known adjacent side, we select the tangent function.
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Figure 5.23, repeated

\[
\tan 22^\circ = \frac{a}{300} \quad \text{Length of side opposite the } 22^\circ \text{ angle}
\]

\[
a = 300 \tan 22^\circ \quad \text{Multiply both sides of the equation by } 300.
\]

\[
a \approx 300(0.4040) \approx 121 \quad \text{Find } \tan 22^\circ \text{ with a calculator in the degree mode.}
\]

The height of the part of the building above the transit is approximately 121 feet. Thus, the height of the building is determined by adding the transit's height, 5 feet, to 121 feet.

\[h \approx 5 + 121 = 126\]

The building's height is approximately 126 feet.

The irregular blue shape in Figure 5.24 represents a lake. The distance across the lake, \(a\), is unknown. To find this distance, a surveyor took the measurements shown in the figure. What is the distance across the lake?

Figure 5.24

If two sides of a right triangle are known, an appropriate trigonometric function can be used to find an acute angle \(\theta\) in the triangle. You will also need to use the inverse key on a calculator. This key uses a function value to display the acute angle \(\theta\). For example, suppose that \(\sin \theta = 0.866\). We can find \(\theta\) in the degree mode by using the secondary inverse sine key, usually labeled \(\sin^{-1}\).

**Study Tip**

\(\sin^{-1}\) is not a button you will actually press. It is the secondary function for the button labeled \(\sin\).

**Many Scientific Calculators:**

\[.866 \quad \text{2nd} \quad \sin^{-1}\]

**Many Graphing Calculators:**

\[2nd \quad \sin^{-1} \quad .866 \quad \text{ENTER}\]

The display should show approximately 59.99, which can be rounded to 60. Thus, if \(\sin \theta = 0.866\), then \(\theta \approx 60^\circ\).
EXAMPLE 9  Determining the Angle of Elevation

A building that is 21 meters tall casts a shadow 25 meters long. Find the angle of elevation of the sun to the nearest degree.

Solution  The situation is illustrated in Figure 5.25. We are asked to find $\theta$. We begin with the tangent function.

$$\tan \theta = \frac{\text{side opposite } \theta}{\text{side adjacent to } \theta} = \frac{21}{25}$$

We use a calculator in the degree mode to find $\theta$.

Many Scientific Calculators:

\[
\left[ \begin{array}{c} 21 \div 25 \end{array} \right] \text{2nd} \text{TAN}\]

Many Graphing Calculators:

\[
\text{2nd} \text{TAN} \left( \begin{array}{c} 21 \div 25 \end{array} \right) \text{ENTER}
\]

The display should show approximately 40. Thus, the angle of elevation of the sun is approximately $40^\circ$.

A flagpole that is 14 meters tall casts a shadow 10 meters long. Find the angle of elevation of the sun to the nearest degree.

Check Point 9

Figure 5.25

The Mountain Man

In the 1930s, a National Geographic team headed by Brad Washburn used trigonometry to create a map of the 5000-square-mile region of the Yukon, near the Canadian border. The team started with aerial photography. By drawing a network of angles on the photographs, the approximate locations of the major mountains and their rough heights were determined. The expedition then spent three months on foot to find the exact heights. Team members established two base points a known distance apart, one directly under the mountain’s peak. By measuring the angle of elevation from one of the base points to the peak, the tangent function was used to determine the peak’s height. The Yukon expedition was a major advance in the way maps are made.