In Exercises 1–4, the graph of a tangent function is given. Select the equation for each graph from the following options:

\[ y = \tan(x + \frac{\pi}{2}), \quad y = \tan(x + \pi), \quad y = -\tan x, \quad y = -\tan(x - \frac{\pi}{2}). \]

In Exercises 5–12, graph two periods of the given tangent function.

5. \( y = 3 \tan \frac{x}{4} \)  
6. \( y = 2 \tan \frac{x}{4} \)  
7. \( y = \frac{1}{2} \tan 2x \)  
8. \( y = 3 \tan 2x \)  
9. \( y = -2 \tan \frac{1}{2} x \)  
10. \( y = -3 \tan \frac{1}{2} x \)  
11. \( y = \tan(x - \pi) \)  
12. \( y = \tan(x + \frac{\pi}{2}) \)  

In Exercises 13–16, the graph of a cotangent function is given. Select the equation for each graph from the following options:

\[ y = \cot(x + \frac{\pi}{2}), \quad y = \cot(x + \pi), \quad y = -\cot x, \quad y = -\cot(x - \frac{\pi}{2}). \]

In Exercises 17–24, graph two periods of the given cotangent function.

17. \( y = 2 \cot x \)  
18. \( y = \frac{1}{2} \cot x \)  
19. \( y = \frac{1}{2} \cot 2x \)  
20. \( y = 2 \cot 2x \)
21. \( y = -3 \cot \frac{\pi}{2} x \)  
22. \( y = -2 \cot \frac{\pi}{4} x \) 
23. \( y = 3 \cot \left( x + \frac{\pi}{2} \right) \)  
24. \( y = 3 \cot \left( x + \frac{\pi}{4} \right) \)

In Exercises 25–28, use each graph to obtain the graph of the reciprocal function. Give the equation of the function for the graph that you obtain.

25. 
\[
\begin{align*}
\text{Graph:} \\
y &= -\frac{1}{2} \sin \frac{1}{2} t \\
x &\in [-4\pi, 4\pi]
\end{align*}
\]

26. 
\[
\begin{align*}
\text{Graph:} \\
y &= 3 \sin 4x \\
x &\in [-\frac{\pi}{8}, \frac{3\pi}{8}]
\end{align*}
\]

27. 
\[
\begin{align*}
\text{Graph:} \\
y &= \frac{3}{2} \cos 2\pi x \\
x &\in [-1, 1]
\end{align*}
\]

28. 
\[
\begin{align*}
\text{Graph:} \\
y &= -3 \cos \frac{3}{2} x \\
x &\in [-4, 4]
\end{align*}
\]

In Exercises 29–44, graph two periods of the given cosecant or secant function.

29. \( y = 3 \csc x \)  
30. \( y = 2 \csc x \) 
31. \( y = \frac{1}{2} \csc \frac{x}{2} \) 
32. \( y = \frac{3}{2} \csc \frac{x}{4} \) 
33. \( y = 2 \sec x \) 
34. \( y = 3 \sec x \) 
35. \( y = \sec \frac{x}{3} \) 
36. \( y = \sec \frac{x}{2} \) 
37. \( y = -2 \csc \pi x \) 
38. \( y = -\frac{1}{2} \csc \pi x \) 
39. \( y = -\frac{1}{2} \sec \pi x \) 
40. \( y = -\frac{3}{2} \sec \pi x \) 
41. \( y = \csc (x - \pi) \) 
42. \( y = \csc \left( x - \frac{\pi}{2} \right) \) 
43. \( y = 2 \sec (x + \pi) \) 
44. \( y = 2 \sec \left( x + \frac{\pi}{2} \right) \)

**Application Exercises**

45. An ambulance with a rotating beacon of light is parked 12 feet from a building. The function 
\[ d = 12 \tan 2\pi t \]
describes the distance, \( d \), in feet, of the rotating beacon from point \( C \) after \( t \) seconds.

a. Graph the function on the interval \([0, 2]\).

b. For what values of \( t \) in \([0, 2]\) is the function undefined? What does this mean in terms of the rotating beacon in the figure shown?

46. The angle of elevation from the top of a house to a jet flying 2 miles above the house is \( x \) radians. If \( d \) represents the horizontal distance, in miles, of the jet from the house, express \( d \) in terms of a trigonometric function of \( x \). Then graph the function for \( 0 < x < \pi \).
47. Your best friend is marching with a band and has asked you to film her. The figure below shows that you have set yourself up 10 feet from the street where your friend will be passing from left to right. If $d$ represents your distance, in feet, from your friend and $x$ is the radian measure of the angle shown, express $d$ in terms of a trigonometric function of $x$. Then graph the function for $-rac{\pi}{2} < x < \frac{\pi}{2}$. Negative angles indicate that your marching buddy is on your left.

In Exercises 48–50, sketch a reasonable graph that models the given situation.

48. The number of hours of daylight per day in your hometown over a two-year period

49. The motion of a diving board vibrating 10 inches in each direction per second just after someone has dived off

50. The distance of a rotating beacon of light from a point on a wall (See the figure for Exercise 45.)

**Writing in Mathematics**

51. Without drawing a graph, describe the behavior of the basic tangent curve.

52. If you are given the equation of a tangent function, how do you find consecutive asymptotes?

53. If you are given the equation of a tangent function, how do you identify an $x$-intercept?

54. Without drawing a graph, describe the behavior of the basic cotangent curve.

55. If you are given the equation of a cotangent function, how do you find consecutive asymptotes?

56. Explain how to determine the range of $y = \csc x$ from the graph. What is the range?

57. Explain how to use a sine curve to obtain a cosecant curve. Why can the same procedure be used to obtain a secant curve from a cosine curve?

58. Scientists record brain activity by attaching electrodes to the scalp and then connecting these electrodes to a machine. The record of brain activity recorded with this machine is shown in the three graphs at the top of the next column. Which trigonometric functions would be most appropriate for describing the oscillations in brain activity? Describe similarities and differences among these functions when modeling brain activity when awake, during dreaming sleep, and during nondreaming sleep.

**Technology Exercises**

In working Exercises 59–62, describe what happens at the asymptotes on the graphing utility. Compare the graphs in the connected and dot modes.

59. Use a graphing utility to verify any two of the tangent curves that you drew by hand in Exercises 5–12.

60. Use a graphing utility to verify any two of the cotangent curves that you drew by hand in Exercises 17–24.

61. Use a graphing utility to verify any two of the cosecant curves that you drew by hand in Exercises 29–44.

62. Use a graphing utility to verify any two of the secant curves that you drew by hand in Exercises 29–44.

In Exercises 63–68, use a graphing utility to graph each function. Use a range setting so that the graph is shown for at least two periods.

63. $y = \tan \frac{x}{4}$

64. $y = \tan 4x$

65. $y = \cot 2x$

66. $y = \cot \frac{x}{2}$

67. $y = \frac{1}{2} \tan \pi x$

68. $y = \frac{1}{2} \tan(\pi x + 1)$

In Exercises 69–72, use a graphing utility to graph each pair of functions in the same viewing rectangle. Use a range setting so that the graphs are shown for at least two periods.

69. $y = 0.8 \sin \frac{x}{2}$ and $y = 0.8 \csc \frac{x}{2}$

70. $y = -2.5 \sin \frac{\pi}{3} x$ and $y = -2.5 \csc \frac{\pi}{3} x$

71. $y = 4 \cos \left(2x - \frac{\pi}{6}\right)$ and $y = 4 \sec \left(2x - \frac{\pi}{6}\right)$

72. $y = -3.5 \cos \left(\pi x - \frac{\pi}{6}\right)$ and $y = -3.5 \sec \left(\pi x - \frac{\pi}{6}\right)$

73. Carbon dioxide particles in our atmosphere trap heat and raise the planet's temperature. The resultant gradually increasing temperature is called the greenhouse effect. Carbon dioxide accounts for about half of global warming. The function

$$y = 2.5 \sin 2\pi x + 0.0216x^2 + 0.654x + 316$$

models carbon dioxide concentration, $y$, in parts per million, where $x = 0$ represents January 1960; $x = \frac{1}{12}$, February 1960; $x = \frac{2}{12}$, March 1960; $\ldots$, $x = 1$, January 1961; $x = \frac{12}{12}$, February 1961; and so on. Use a graphing utility to graph the function in a $[30, 45, 5]$ by $[310, 420, 5]$ viewing rectangle. Use a range setting so that the graphs are shown for at least two periods.
For what effect does it have on the graph of $y = \sin \frac{1}{x}$? What kind of behavior can be modeled by a function such as $y = 2^{-x} \sin x$?

**Critical Thinking Exercises**

In Exercises 75–76, write an equation for each blue graph.

75. 

Graph in a viewing rectangle. Describe what the graph reveals about carbon dioxide concentration from 1990 through 2005.

74. 

Graph $y = \sin \frac{1}{x}$ in a $[-0.2, 0.2, 0.01]$ by $[-1.2, 1.2, 0.01]$ viewing rectangle. What is happening as $x$ approaches 0 from the left or the right? Explain this behavior.

76. 

For $x > 0$, what effect does $2^{-x}$ in $y = 2^{-x} \sin x$ have on the graph of $y = \sin x$? What kind of behavior can be modeled by a function such as $y = 2^{-x} \sin x$?