Angles and Their Measure

Angles
An angle is formed by two rays that have a common endpoint called the vertex. One ray is called the initial side and the other the terminal side. The arrow near the vertex shows the direction and the amount of rotation from the initial side to the terminal side.

Angles of the Rectangular Coordinate System
An angle is in standard position if
• its vertex is at the origin of a rectangular coordinate system and
• its initial side lies along the positive x-axis.
Measuring Angles Using Degrees

The figures below show angles classified by their degree measurement. An acute angle measures less than 90°. A right angle, one quarter of a complete rotation, measures 90° and can be identified by a small square at the vertex. An obtuse angle measures more than 90° but less than 180°. A straight angle has measure 180°.

\[ \theta \]

Acute angle \[ 0^\circ < \theta < 90^\circ \]

Right angle \[ 1/4 \text{ rotation} \]

Obtuse angle \[ 90^\circ < \theta < 180^\circ \]

Straight angle \[ 1/2 \text{ rotation} \]

Coterminal Angles

An angle of \( x^\circ \) is coterminal with angles of \( x^\circ + k \cdot 360^\circ \)

where \( k \) is an integer.

Assume the following angles are in standard position. Find a positive angle less than 360° that is coterminal with:

a. a 420° angle
b. a -120° angle.

Solution

We obtain the coterminal angle by adding or subtracting 360°. Our need to obtain a positive angle less than 360° determines whether we should add or subtract.

a. For a 420° angle, subtract 360° to find a positive coterminal angle.

\[ 420^\circ - 360^\circ = 60^\circ \]

A 60° angle is coterminal with a 420° angle. These angles, shown on the next slide, have the same initial and terminal sides.
Solution
b. For a \(-120^\circ\) angle, add 360° to find a positive coterminal angle.
\[-120^\circ + 360^\circ = 240^\circ\]
A 240° angle is coterminal with a \(-120^\circ\) angle. These angles have the same initial and terminal sides.

Finding Complements and Supplements
• For an \(x^\circ\) angle, the complement is a 90° \(- x^\circ\) angle. Thus, the complement’s measure is found by subtracting the angle’s measure from 90°.
• For an \(x^\circ\) angle, the supplement is a 180° \(- x^\circ\) angle. Thus, the supplement’s measure is found by subtracting the angle’s measure from 180°.

Definition of a Radian
• One radian is the measure of the central angle of a circle that intercepts an arc equal in length to the radius of the circle.
Radian Measure

Consider an arc of length $s$ on a circle of radius $r$. The measure of the central angle that intercepts the arc is

$$\theta = \frac{s}{r} \text{ radians.}$$

Conversion between Degrees and Radians

- Using the basic relationship $\pi \text{ radians} = 180^\circ$.
- To convert degrees to radians, multiply degrees by $\frac{\pi \text{ radians}}{180^\circ}$.
- To convert radians to degrees, multiply radians by $\frac{180^\circ}{\pi \text{ radians}}$.

Example

Convert each angle in degrees to radians

$40^\circ$
$75^\circ$
$-160^\circ$
Example cont.

Solution:
• $40^\circ = \frac{40\pi}{180} = \frac{2\pi}{9}$
• $75^\circ = \frac{75\pi}{180} = \frac{5\pi}{12}$
• $-160^\circ = \frac{-160\pi}{180} = \frac{-8\pi}{9}$

Length of a Circular Arc

Let $r$ be the radius of a circle and $\theta$ the non-negative radian measure of a central angle of the circle. The length of the arc intercepted by the central angle is

$$s = r \theta$$

Example

A circle has a radius of 7 inches. Find the length of the arc intercepted by a central angle of $2\pi/3$

Solution:

$$s = (7 \text{ inches}) \times (\frac{2\pi}{3})$$

$$= 14 \frac{\pi}{3} \text{ inches}$$
Definitions of Linear and Angular Speed

If a point is in motion on a circle of radius $r$ through an angle of $\theta$ radians in time $t$, then its linear speed is

$$v = \frac{s}{t}$$

where $s$ is the arc length given by $s = r \, \theta$, and its angular speed is

$$\omega = \frac{\theta}{t}$$

Linear Speed in Terms of Angular Speed

• The linear speed, $v$, of a point a distance $r$ from the center of rotation is given by $v = r \, \omega$ where $\omega$ is the angular speed in radians per unit of time.

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