Definitions of the Trigonometric Functions in Terms of a Unit Circle

- If \( t \) is a real number and \( P = (x,y) \) is a point on the unit circle that corresponds to \( t \), then

\[
\sin t = y \quad \cos t = x \quad \tan t = \frac{y}{x} \quad \sec t = \frac{1}{x} \quad \csc t = \frac{1}{y} \quad \cot t = \frac{x}{y}
\]

Text Example

Use the figure at the right to find the values of the trigonometric functions at \( t = \pi/2 \).

Solution: The point \( P \) on the unit circle that corresponds to \( t = \pi/2 \) has coordinates \((0, 1)\). We use \( x = 0 \) and \( y = 1 \) to find the values of the trigonometric functions.

\[
\sin \frac{\pi}{2} = 1, \quad \cos \frac{\pi}{2} = 0 \\
\csc \frac{\pi}{2} = \frac{1}{1} = 1, \quad \cot \frac{\pi}{2} = \frac{0}{1} = 0
\]

By definition, \( \tan \frac{\pi}{2} \) and \( \sec \frac{\pi}{2} \) are undefined.
The Domain and Range of the Sine and Cosine Functions

- The domain of the sine function and the cosine function is the set of all real numbers. The range of these functions is the set of all real numbers from –1 to 1, inclusive.

Even and Odd Trigonometric Functions

- The cosine and secant functions are even.
  \[ \cos(-t) = \cos t \quad \sec(-t) = \sec t \]

- The sine, cosecant, tangent, and cotangent functions are odd.
  \[ \sin(-t) = -\sin t \quad \csc(-t) = -\csc t \]
  \[ \tan(-t) = -\tan t \quad \cot(-t) = -\cot t \]

Example

- Find the exact value of \(\sin(-45^\circ)\)

\[ \sin(-45) = -\sin(45) = -\frac{\sqrt{2}}{2} \]
Definition of a Periodic Function

• A function $f$ is periodic if there exists a positive number $p$ such that
  • $f(t + p) = f(t)$
  • For all $t$ in the domain of $f$. The smallest number $p$ for which $f$ is periodic is called the period of $f$.

Periodic Properties of the Sine and Cosine Functions

• $\sin(t + 2\pi) = \sin t$ and $\cos(t + 2\pi) = \cos t$
• The sine and cosine functions are periodic functions and have period $2\pi$.

Example

• Find the exact value of $\cos(5\pi)$
  
  $\cos(5\pi) = \cos(4\pi + \pi)$
  
  $= \cos \pi = -1$
Periodic Properties of the Tangent and Cotangent Functions

• tan \((t + \pi) = \tan t\) and cot \((t + \pi) = \cot t\)

• The tangent and cotangent functions are periodic functions and have period \(\pi\)

Repetitive Behavior of the Sine, Cosine, and Tangent Functions

For any integer \(n\) and real number \(t\), is
\[\sin (t + 2\pi n) = \sin t, \cos (t + 2\pi n) = \cos t,\]
and \[\tan (t + \pi n) = \tan t.\]